Incentives to Correct Misbehaviour: When Less is Better *

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Abstract

Can incentives promote the sustainability of good habits in the long-term? We analyze the optimal pricing policy that a paternalistic social planner should impose to correct the behaviour of time-inconsistent consumers. Myopic consumers can reduce their future expenditure by participating in programs promoting virtuous behaviour and by exerting self-control effort. We find that incentives may be ineffective in correcting misbehaviour both when myopia is low and when it is high, despite the complementarity between participation and effort decisions. Moreover, optimal incentives are higher for naive consumers when myopia is high, and for sophisticated ones when myopia is low.

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JEL codes: D11, D91

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1 Introduction

In March 2010, the US Patient Protection and Affordable Care Act introduced the possibility for employers to vary the health insurance premium charged to employees by as much as 30 percent, on condition that certain health status factors, such as body mass index, tobacco use, physical fitness or activity levels, were met. In 2015, such workplace wellness programs were offered by 57% of US employers (Kaiser Family Foundation, 2015), and by more than 80% of those with more than 1,000 employees (Mattke et al., 2015). Despite the substantial size of financial incentives, the uptake of worksite wellness programs remained limited, with participation rates below 50% of the eligible employees (Mattke et al., 2015; Persson et al., 2013). In addition, even when employees did participate, the success of such programs was disappointing. In a year-long randomized controlled trial run on 197 obese employees at the University of Pennsylvania, a financial reward of $550 for having achieved a weight loss goal did not produce any significant effect on weight reduction (Patel et al., 2016)[1].

In the UK, more than one out of three Clinical Commissioning Groups are currently denying or delaying routine surgery under the National Health Service to smokers and obese patients (Royal College of Surgeons, 2016), thereby promoting long-term behaviour changes by raising the access cost to healthcare services. These controversial policies have been questioned not only on the grounds of discriminatory, ethical and clinical arguments. Doubts have also been cast about their effectiveness in actually tackling problem behaviour[2].

Incentives are increasingly being introduced by policymakers to motivate people to change their behaviour, often as part of schemes aimed at reducing obesity, smoking, alcoholism and other harmful habits. However, the evidence so far shows that people seem hesitant to participate in programs that involve long-term behavioural changes, even when such programs are heavily incentivised. Moreover, when they do participate in such pro-

[1]Similar evidence can be observed for the repeated failed attempts of smokers to stop smoking (Pallonen et al. 1990; Chassin et al. 1996). A national survey conducted in England in 2010 revealed that 63% of regular 11-15 year old smokers had unsuccessfully tried to stop smoking, although they generally smoked less and were more sensitive to cigarette prices than older smokers (Fuller, 2011).

[2]In October 2017, Robert West, professor of health psychology at UCL, commenting on the Hertfordshire CCG decision to make the ban indefinite for their patients, stated that "Rationing treatment on the basis of unhealthy behaviours betrays an extraordinary naivety about what drives those behaviours", see http://edition.cnn.com/2017/10/31/health/smokers-obese-no-surgery-nhs-uk/index.html.

grams, their efforts diminish over time: people seem unable to follow up on their good intentions when long-term horizons are involved.

The inconsistency of individuals' behaviour over time raises some important questions. What is the role of financial incentives in fostering the uptake of programs *and* the subsequent efforts? Why do they partially fail? And what is the optimal level of incentives, when accounting for the social cost of such programs? In this paper, we address these questions, which inherently are of great interest for policymakers.

From a theoretical point of view, the disappointing evidence on participation rates and program failures is particularly puzzling. In principle, the rate of program uptake and the probability of successfully concluding one are expected to be positively correlated, as incentives motivate both participation and effort. The correlation should be even stronger if we account for the positive cross-effects that exist between the two: the expectation of higher efforts should induce higher participation rates. Notwithstanding the financial incentives, the inexplicably low participation and success rates suggest that the interaction between participation and effort decision could be more tangled than one usually believes. This paper is an attempt to shed light on this interplay between intertemporal behaviour and incentives in the presence of time-inconsistent consumers.

From a policymaking point of view, the fact that consumers' myopia leads to misbehaviour, rather than to a misconsumption decision, has important implications, in terms of welfare effects of the incentive schemes. The crux of the problem is that habits, unlike consumption, are not marketed. In the absence of a market price, it is not possible to directly impose a tax/subsidy on efforts. Moreover, in practice, the application of direct punishment or reward mechanisms, conditional to the outcome of behaviour, is not feasible on a large scale or outside the realm of experiments, either because its outcome is not observable, or because these mechanisms would be discriminatory. For instance, the excessive consumption of unhealthy food could easily be discouraged by introducing taxes on it, while the insufficient practice of physical activity cannot be punished directly in real contexts. Policymakers can only encourage efforts in this direction, for example by increasing the price of healthcare services. The use of direct rather than indirect forms of incentives leads to profound welfare consequences. In the case of misconsumption (which is the focus of the literature on sin taxes), tax proceeds can be returned, in a fixed form, to consumers, and no welfare loss occurs if consumers are homogeneous in their myopia (O'Donoghue and Rabin, 2003). However, we show that, in the case of misbehaviour, some welfare loss is unavoidable, as the correction of misbehaviour necessarily requires the distortion of the consumption of other goods. As a consequence, incentives to correct myopic behaviour decisions are always socially costly, and policymakers need to determine their optimal level by trading-off between the social cost of incentives and their effectiveness.

In our model, we focus on the non-marketed component of behaviour, which is neither observable nor contractible. Because of these characteristics, only indirect (and socially costly) incentives, in the form of a price distortion of goods whose consumption is affected by the consumer's actions, can be applied to correct behaviour.

We assume that a myopic consumer can exert quantity-saving efforts, which allow her to achieve a certain level of utility, but with lower future consumption, hence with lower expenditure. However, in order to reap any benefit, the efforts must be exerted consistently over two different periods, i.e. the efforts are complementary. The first-stage effort can be seen as being related to a participation decision, while the second-stage effort can be interpreted as being related to self-control decisions. For instance, the effort of joining a worksite wellness program and then actually attending it would lead to future savings in healthcare costs. Given that the benefit of the two efforts, in terms of savings, is accrued in the future, while its cost is paid in advance, a present-biased consumer misbehaves by underexerting her efforts in both participation and in self-control activities. In order to induce the consumer to improve her behaviour, a paternalistic social planner should increase the value of the future benefits. This is achieved by raising the price of the goods whose consumption is reduced by the effort. Therefore, a trade-off takes place between the deadweight loss of the rise in price and the positive effect it has on correcting misbehaviour.

Our analysis points out to what extent the level of incentives should be raised as a function of the levels of consumer's myopia and the awareness of her present-bias problem. We identify the drivers that underlie the consumer's decision and provide a theory that offers insights into the existing theoretical puzzle and empirical evidence.

We find that myopia affects the effectiveness of incentives over three different dimensions.

First, a very myopic consumer has actually more to gain from an increase in self-control or in participation efforts, but it is more difficult to convince her to exert these efforts, as the benefits can only be collected in the future, which she values less because of the strong present bias. As a consequence, policies meant to improve the behaviour of very myopic consumers are less effective. A myopic consumer is less responsive to this kind of incentive. Second, even though a very myopic person can be convinced to participate, she will later refrain from actually exerting efforts, thus wasting the cost of participation. Therefore, not only is it more difficult to convince her to participate, but, on top of this, the usefulness of incentives to participate is dissipated by the future misbehaviour. Third, highly myopic consumers display low participation rates, thus implying that the value of any additional effort is dissipated by their lack of willingness to participate. The reduced reach of incentive campaigns discourages the implementation of policies directed at inducing self-control efforts.

Overall, these three factors explain the lack of effectiveness of incentive policies that address myopic consumers. Despite the fact that these consumers display severe misbehaviour with high social costs, policymakers might want to target these consumers less aggressively than less myopic individuals. On the other hand, we find that incentives should also be low when consumer's myopia is weak, as social inefficiency is less severe.

The trade-off between value and effectiveness of incentives has important policy implications, as it suggests that the optimal size of incentives should be non-monotonic in the degree of myopia. In particular, it should be low both when myopia is low (to account for the low marginal value of further efforts and participation) and when it is high (to account for the consumer's unresponsiveness, limited reach of the policy and high drop-out rates).

Interestingly, we also find that the level of incentives does not necessarily reflect the virtuous circle generated by the complementarity between participation and self-control. In fact, if myopia is sufficiently weak, the simultaneous increase in effort in self-control and participation induced by a lower degree of myopia reduces the inefficiency to the point that incentives should be reduced, rather than increased, to exploit and foster the interplay between self-control and participation.

The effectiveness of incentives is also affected to a great extent by the consumer's awareness of her own degree of myopia. We distinguish between *sophisticated consumers*, who are well aware of their myopia, and *naive ones*, who conversely do not anticipate their self-control problems (see Della Vigna and Malmendier, 2004; O'Donoghue and Rabin, 2003). Naive consumers overestimate their future ability to exert self-control, and are hence more likely to sink the initial participation cost. This has both positive and negative effects on the optimal level of incentives. On the one hand, a policy that incentivises self-control has a larger outreach with more naive consumers, i.e. incentives are socially more valuable in terms of effects on short-term behaviour (hence, they should be increased); however, the under-participation problem is less severe for naive consumers, i.e. incentives are socially less valuable in terms of effects on long-term behaviour (hence they should instead be lowered).

This work contributes to the debate on the optimal policy that should be introduced in the presence of time-inconsistent consumers, and it is related to two main strands of literature, which correspond to the two types of instruments (financial and not) that are available for policymakers to correct consumers' behaviour.

The first strand of literature investigates the economic incentives, mainly in the form of sin taxes applied to consumption goods (see, e.g., O'Donoghue and Rabin, 1999a, 1999b, 2003 and 2006; Haavio and Kotakorpi, 2011). Since the consumer's utility depends on the quantity alone, raising the price (and then returning the proceeds to the consumer by means of a fixed subsidy) fully corrects the distortion in the consumption decision caused by the consumer's present bias and the first best level can be implemented. We add to this literature by modelling consumers' behaviour. We do this by introducing an effort that could be exerted before the consumption decision, and which has an impact on the demand for the goods. A typical result in the literature on sin taxes is that if all consumers are affected by self-restraint problems to the same degree, then a price distortion allows the first best to be achieved, with no welfare loss. We show that this is no longer true in a framework in which the consumer's mistake pertains to behaviour. A similar conclusion was reached by Lerner (1970) and Dixit (1970), when studying optimal indirect taxes when one sector was not taxable (as in the case of effort). However, their focus was on the optimal allocation of taxes between the taxable and non-taxable sectors, while ours is related to the impact of incentives on the consumer's behaviour in the long-term.

The second strand of literature pertains to the non-pricing instruments whose aim is to reduce a consumer's choice set (Strotz, 1955; Gul and Pesendorfer, 2001; Ashraf et al., 2006; Laibson, 2015). Their purpose is to curb, from the outset, any possibility of succumbing to the temptation of misbehaviour. The social planner must then induce the consumer to commit to a restricted set of alternatives. The problem with commitment is that it is a problematic prediction, as very little of it can be observed in the economy. A growing amount of experimental literature (see, e.g., Giné et al., 2010; Kaur et al., 2015; Laibson, 2015) finds that only a few subjects decide to tie their own hands, and they rarely express a willingness to pay to have their choice-set reduced. While commitment and participation can be seen as opposite phenomena (the former restricting the choice set, the second enlarging it), they actually represent two sides of the same coin, as they both entail a decision that changes the set of future alternatives. In fact, in our model, the participation decision could be symmetrically seen as a commitment not to misbehave in the future.

This work can be considered as an attempt to bridge the gap between the literature on economic incentives and that on commitment. In fact, we look into the economic incentives that induce a consumer to commit to a situation in which the possibilities of harmful behaviour are diminished.

The rest of the paper is organised as follows. In Section 2, a general model is introduced to present the central idea and relate it to the literature on sin taxes. In Section 3, we introduce the model and characterise the social optimum, i.e. our benchmark, in Section 4. Section 5 analyses the socially optimal price that is necessary to induce a time-inconsistent consumer to invest in an effort. In particular, we identify how incentives drive the participation and self-control decisions. Section 6 examines the role of consumer's myopia and how it affects the effectiveness of incentives on participation and self-control. Section 7 studies the role of consumer's awareness on incentive design and Section 8 presents the conclusions.

All the proofs and technical details are relegated to Appendix 1.

2 The economic implications of misbehaviour: a general framework

In this section, we explore the main economic implications of the fact that a consumer's present bias pertains to a behaviour decision, rather than to a consumption decision. We adopt the same framework that is generally used in the literature on sin goods (O'Donoghue and Rabin, 2003, 2006; Haavio and Kotakorpi, 2011). We then focus on the results obtained from this literature on the welfare effects of the incentives introduced to correct a biased consumption decision. Finally, we analyse the situation in which the consumer's present-bias affects behaviour, i.e. an effort. We show that this assumption has clear implications on both the type of incentives and on their social cost.

2.1 Incentives and welfare effects in the case of misconsumption

We draw from the standard literature on sin taxes (see, e.g., O'Donoghue and Rabin, 2003) and initially consider a simple consumption model with two generic goods, whose quantities are denoted with x and y. For simplicity, we assume that the production of both goods entails constant returns to scale; we denote the marginal costs of x and y with c and d, respectively. The consumer's utility is expressed by the function $V(x, y)$, with $V_x, V_y > 0$, $V_{xx}, V_{yy} < 0$[3]. Although the utility provided by x is concurrent with its consumption, part of the utility provided by y is obtained in the future. For example, y could be the hours of gym attendance that lead to a future benefit, in terms of improved health, and x is the numeraire.

If there is no time-consistent discount factor, a social planner maximizes the consumer's net surplus $V(x, y) - cx - dy$ by choosing x^* so that $V_x = c$ and y^* so that $V_y = d$.

Our focus is on the case of a myopic consumer who is biased about the present over the future. Since part of the benefits from the consumption of y are reaped in the future, the utility provided by y is underrated by the consumer's present self, thus generating a

[3] In the literature on sin taxes, the utility function is usually assumed separable. For example, O'Donoghue and Rabin (2003) assumed that $V(x, y, Z) = \rho \ln x - \gamma \ln y + f(Z)$, where $\rho, \gamma > 0$ are exogenous parameters and Z is the vector of other goods; Haavio and Kotakorpi (2011) assumed that $V(x, y, z) = v(x) - h(y) + z$, where z is a composite good.

distortion between the actual and optimal consumption. We model this by assuming that the consumer chooses her consumption by maximizing function $V(x, \beta y) - p_x x - p_y y$, where the constant coefficient $\beta < 1$ discounts the utility provided by y, and p_x and p_y denote the prices of the two goods. The time-inconsistent consumer chooses the quantities x^{inc} and y^{inc}, so that $V_x = p_x$ and $V_y = p_y/\beta$. From the comparison between (x^{inc}, y^{inc}) and (x^*, y^*), it is straightforward to see that the social planner can realign the consumer's problem to the first best and achieve the optimal consumption by setting $p_x^* = c$ and $p_y^* = \beta d$, namely by imposing a distortion of the price of the good whose demand is affected by the consumer's present bias.

In short, if $\beta = 1$ (i.e., the consumer is not present biased), the optimal price is at the marginal cost. Conversely, if $\beta \neq 1$, a tax/credit is applied to price p_y to correct the distortion of its consumption. Regardless of the level of myopia, and provided that the consumers are homogeneous in β, the first best can always be achieved.

2.2 Modelling behaviour

The above discussion focused on a specific category of consumers' choices, which are related to purchasing activities. In fact, the goods x and y were both acquired at a market price.

However, in many cases, utility also depends on actions that require minimal or no financial outlay and which are not marketed, due to excessively high monitoring costs. These behaviours concern the everyday habitual elements of an individual's lifestyle, and their cost is mainly related to the efforts adopted to avoid wasteful actions or to engage in constructive endeavours. Such actions are beneficial because, by paying attention to one's behaviour, it is possible to achieve a greater utility, given the same levels of consumption. For example, by exercising regularly or attending to one's studies one obtains long-term benefits in terms of improved health or better job conditions[4].

Let us suppose that y represents a non contractible behaviour rather than physical goods. For example, y is the effort necessary to exercise regularly, which allows us to improve our

[4] Applications abound. In the energy field, we can reduce a future bill by exerting efforts in such behaviours as shutting the windows when the air conditioning is on, using a clothes line rather than a tumble drier, putting a full load of washing on rather than a half load. Other examples can also be found in the literature on self-control or procrastination (O'Donoghue and Rabin, 1999a, 1999b, 2008).

health (i.e., utility), given the same level of consumption x. As an effort, y is not traded on the market, and it does not have a market price, but only a cost d; in order to make our point and maintain the parallelism with the standard model presented above, we assume in this section a linear effort cost[5].

In this framework, the quantities (x^*, y^*) are still the solution to the social planner's problem. However, the consumer now maximizes $V(x, \beta y) - p_x x - dy$. The effort is not contractible, hence no incentive can be conditioned to the level of y.

The consumer chooses x^{inc} and y^{inc} so that $V_x(x^{inc}, y^{inc}) = p_x$ and $V_y(x^{inc}, y^{inc}) = d/\beta$. When $V_{xy}(\cdot) \neq 0$, the price p_x affects not only the choice of x^{inc}, but also that of y^{inc}. As a consequence, when $V_{xy}(\cdot) \neq 0$, a trade off emerges between correcting the inefficient behaviour y^{inc} and causing an inefficient consumption x^{inc}. Conversely, when $V_{xy}(\cdot) = 0$, correcting behaviour through prices by exploiting the cross-elasticities of demand is impossible, so that there is no point in applying a tax/subsidy to the consumption of x^{inc}, and a welfare loss cannot be avoided. The first best cannot be achieved in either case ($V_{xy}(\cdot) \neq 0$ or $V_{xy}(\cdot) = 0$), despite the fact that consumers are homogeneous in their myopia[6].

From a policymaking perspective, the most interesting case is the former, where the two marginal utilities are not independent, as this situation leaves room for the application of incentive schemes. In this case, pricing policies have to be carefully calibrated to trade off an inefficient behaviour and an inefficient consumption.

The welfare effects of a price distortion have been studied extensively ever since Ramsey (1927) first raised the question of optimal taxation. In the literature on indirect taxes (Lerner, 1970; Baumol and Bradford, 1970; Diamond and Mirrlee, 1971; Atkinson and Stiglitz, 1972, 1976), the focus is on the optimal tax applied to different goods or sectors. In some respects, our work draws on this literature, in that we account for the deadweight loss of a higher price. However, we depart from this literature in two important ways. First, in the literature on

[5]We remark once more that y is a pure effort, net of any price effects; in our example on healthy exercise, this means that y is the effort that we need to exert, *in addition* to the fact that we pay an entry ticket to the gym.

[6]It should be noted that the inefficiency originates from the biased proportion of x and y, which in turn results from the solution to the consumer's problem. Therefore, it cannot be solved by adopting a different tariff structure for x. In fact, even if a two-part tariff were applied, instead of the per unit price p_x, the fixed component of the tariff would have no impact on the combination of x and y

indirect taxation, price distortion is necessary to raise some fixed revenues from taxes, due to the imposition of an exogenous constraint. Conversely, the price distortion in our model is necessary to correct a myopic effort. Therefore, the surplus is affected not only by the deadweight loss, but also by the endogenous effect of the price on the consumer's behaviour, which shifts the demand function. Second, when a long-term horizon is considered, in the case of goods a social planner could impose differentiated prices for any period, in order to address, and possibly resolve, any intertemporal externality (see, e.g., Beshears et al., 2005). Conversely, when the effort is not contractible, incentives cannot separately target efforts exerted at different times. Therefore, some intertemporal externality may arise, so that short-term incentives interact with the long-term ones. The focus of this paper is the interplay between the effects of behaviours exerted over different periods. We show that such an interaction has important policy implications on the design of the incentives for participation and future behaviour.

2.3 A foreword on alternative regulatory tools

This paper studies how indirect taxes on the consequences of misbehaviour can provide incentives to correct it. Before proceeding with the analysis, a natural question arises: is it not possible to correct misbehaviour with instruments other than taxes or prices or, more in general, financial incentives?

The answer to the question is surely positive. A government could try to steer consumers' behaviour by raising the direct cost of misbehaviour, or by reducing the utility it provides. For example, smoking regulations reduce the consumers' utility by forcing them to smoke outdoors. Other measures, such as bans or restrictions, prevent misbehaviour from the outset.

While all these measures are surely successful to some extent, they are also unable to completely resolve the problem of consumers' misbehaviour. First, they can prevent misbehaviour in some situations or for some categories of individuals, but they cannot eradicate it completely. For example, even when schools are forbidden to provide junk food, children can still consume it at their own homes. Obliging people to smoke outdoors can certainly discourage smoking in public places, but this ban cannot be enforced in private homes.

Second, banning an activity does not *per se* prevent people from doing it. The effectiveness of the policy depends on the efficacy of the monitoring activities, which is not perfect in real situations. In some cases, monitoring is completely impossible: social planners have no control over the amount of exercise people do, what they eat or how much they smoke at home.

The empirical evidence on the effectiveness of policy measures other than taxes is in fact mixed (Stehr, 2007; Bernheim, Meer and Novarro, 2016; Carpenter and Eisenberg, 2009). In addition, these measures are not without their own social cost. In other words, bans or other forms of commitment in practice restrict the choice set, thus causing a disutility for consumers (Gul and Pesendorfer, 2001; Beshears et al., 2005), to the point that consumers show less willingness to pay to commit (Laibson, 2015; Augenblick et al., 2014). Even sales restrictions only work if they provide a commitment device to time inconsistent consumers; however, they also increase the inconvenience of time consistent consumers, who simply buy in advance. Hinnosaar (2016) has empirically shown that sales restrictions may not perform as well as taxes if consumers are heterogeneous in terms of consumer welfare.

In conclusion, regulatory measures, such as bans or sales restrictions, can certainly alleviate the problem of consumers' misbehaviour. However, by constraining the consumer's choice, these measures might entail social costs if the consumers display a certain form of heterogeneity. Even more importantly for our purposes, they are not able to fully eradicate the problem in most cases, and some residual amount of misbehaviour persists. In short, all behaviours that are not observable and non contractible fall beyond the outreach of any direct regulatory intervention of this nature.

The focus of this work is this specific component of misbehaviour, that is, the one that is not observable and non contractible. Translated into the terms of our model, the effort variable y is normalised to represent only the specific component of behaviour that is non contractible and therefore cannot be addressed by direct regulatory instruments such as bans, caps, floors, monitoring activities and incentives applied directly to the contractible component of effort. The only way of discouraging non observable misbehaviour is through indirect incentive measures, by making consumers voluntarily refrain from it, such as by raising the cost of its consequences.

3 A simple model

In this section, we set up a three-period model. The three stages are necessary to study the participation, self-control effort and consumption decisions, respectively[7]. We allow consumers to be time-inconsistent and (possibly) overconfident. The aim is to find the welfare-maximizing price that provides the right incentives to guide consumer's behaviour. In particular, we study the role of the price of the consumption good on i) the incentives to participate (i.e. on the long-term behaviour), ii) the incentives to exert a self-restrain effort (i.e. on the short-term behaviour), and iii) the interaction between the short- and long-term incentives.

Owing to the complex interaction of the forces at play, in order to make our point in the most straightforward way, we focus on the case in which the efforts and consumption are substitutes[8].

A market is characterised by the presence of a single representative consumer. In choosing her level of consumption, the consumer seeks to maximize her utility $V(s)$, which is a function of the level of service s provided by the good that she purchases[9], with $V_s > 0$ and $V_{ss} < 0$. The consumer's level of service is a function of the consumed quantity x and of the self-control effort y to increase the efficiency of her consumption, so that $s = x + y$. The self-control effort allows the consumer to achieve the same level of service s with a lower consumption x. In particular, one additional unit of effort allows the consumer to save one unit of consumed quantity. For example, s is the level of future health, x is the quantity of purchased healthcare services and y is the effort exerted to achieve healthy behaviour (exercising regularly, having a healthy diet, etc.) that allows one to save on healthcare costs[10]. The self-control effort is

[7]The consumer's decisions are thus taken sequentially in order to reflect the actual decision making problem more closely; however, it should be noted that the model can be solved equivalently as if the consumer's choice were made simultaneously in the initial planning stage, as the decision maker is the same in all the periods.

[8]This assumption is made with no loss of generality. See footnote 13.

[9]For a similar approach, based on the consumer's level of service – comfort– applied to energy efficiency, see Chu and Sappington (2013).

[10]The model could also be applied with few, if any, adaptations, to the education case (studying assiduously

neither observable nor contractible, and it entails an increasing cost $d(y)$. We assume that $d(0) = 0$ (the effort entails no fixed cost), $d_y > 0$ (the effort is costly), $d_{yy} > 0$ (the cost of the effort is convex), and, for technical reasons, $d_{yyy} \geq 0$ (which is a sufficient condition for a social planner's problem to be concave, see also Laffont and Tirole, 1993). The self-control effort can only be exerted if the consumer paid a participation cost k at an earlier stage (e.g., a sort of participation effort in a wellness program): if the consumer fails to pay k, she is in practice committing to misbehaving in the future. This implies that participation and self-control effort decisions are complementary: to actually achieve some savings, the consumer must invest consistently in good behaviour over time. The value of k is perfectly observed by the consumer, while the social planner only knows its distribution. For simplicity, we assume that k is uniformly distributed over the interval $[0, K]$, with $K > 0$[11].

If the consumer has committed not to exert a self-control effort, the level of service is $s = x$. Conversely, if the consumer has paid the participation cost k, she later has the possibility of exerting y, and the level of service is given by $s = x + y$.

The production of the good entails a constant marginal cost c, which also includes the social cost imposed on society. The assumption of constant marginal costs is made with no loss of generality concerning our main results, but it significantly streamlines the analysis.

A social planner charges a uniform price p for the consumption x[12]. Given the price, the consumer first decides on the self-control effort y, and then on the quantity x to purchase; on the basis of these decisions, she achieves the level of service $s = x + y$.

The timing of the game is as follows:

1. in $t = 0$, the social planner announces the price p and the consumer privately observes k. The consumer then decides whether to commit and, depending on her decision, pays the commitment cost k, or not;

to gain a better income) or to the energy sector (pooling car trips with colleagues to save fuel, switching off the lights whenever one leaves a room or using the washing machine at full capacity so as to reduce the energy bills by lowering the consumption of electricity or water).

[11]We will later show that the heterogeneity of consumer types with respect to k is not the cause of inefficiency. In fact, we will verify that the deadweight loss originates from misbehaviour, i.e. the effort bias, which compels the use of indirect incentives.

[12]In this framework, a fixed fee would merely have redistributive purposes, by returning the proceeds of the production to consumers; therefore, we will focus on the optimal per-unit price (the same approach is used in the literature on sin taxes; see, e.g., O'Donoghue and Rabin, 2003 and 2006).

2. in $t = 1$, the consumer decides on a level of effort $y \geq 0$ and pays the effort cost $d(y)$, if she previously paid k; if she did not pay k, $y = 0$;

3. in $t = 2$, the consumer decides on the quantity consumed, x, and receives the net surplus $V(s) - px$ from the consumption.

The social planner seeks to maximize welfare, W, defined as the present value of the consumers' utility net of production and effort costs: $W = \delta\,(V(s) - cx) - d(y)$, where $\delta \leq 1$ is the time-consistent discount factor and y is the effort exerted in $t = 1$, conditional to the participation decision of $t = 0$.

The first best policy is found in Section 4. In Section 5, we introduce the assumption of myopic behaviour of the consumer and analyse its impact on the welfare-maximizing policy.

Before proceeding with the analysis, it may be worth dedicating a few words to the timing of the model. The reasons for introducing a three-stage game, that is, for separating the participation, effort and consumption decisions, are manifold.

First, in many real situations, a person's behaviour is conditioned by decisions to which they committed in the past. For example, a person can only attend to their studies if they previously enrolled at a university. By extending this argument, it is possible to say that the benefits of good behaviour are often the result of a continued effort that is consistently exerted over a long time.

Second, the decision of participating depends on the expectation about one's future behaviour. A person does not apply to a university or join a gym if they know that they are not going to attend. Therefore, introducing a participation stage allows one to study the role of the consumer's awareness of her own myopia (O'Donoghue and Rabin, 2001), and how this awareness affects the level of incentives necessary to correct the present-bias.

Finally, the introduction of an initial stage to the game, in which the consumer decides to commit to a future effort, allows one to gain some (rather unexpected) insights into the interaction between the incentives on the short-term decision of exerting an effort and the incentives on the long-term decision of participating.

4 Benchmark: time-consistent consumers

The social planner's problem is solved backwards. In $t = 2$, given the effort y exerted in $t = 1$, the price p and the participation decision of $t = 0$, the quantity x demanded by a consumer maximizes the function $V(s) - px$. The demand x is thus expressed by the FOC $V_s(s) = p$. Thus, the consumer achieves a level of service

$$s^* = V_s^{-1}(p). \tag{1}$$

It should be noted, from (1), that a higher price reduces the achieved level of service s (as $V(s)$ is concave in s). In turn, a lower level of service implies a lower gross surplus $V(s)$.

From (1), the consumer's demand for x is

$$x^* = V_s^{-1}(p) - y \tag{2}$$

given $y \geq 0$ exerted in the previous stage. From (2), the demanded quantity is a decreasing function of both the price and y, i.e. the self-control effort. In particular, equation (2) clearly shows that y represents the quantity saved due to the effort for a given price p.

In $t = 1$, and provided that the consumer paid k in the previous stage, she chooses the effort y by maximizing $\delta[V(s^*) - px^*] - d(y)$, i.e., by using (1) and (2),

$$\max_y \delta[V\left(V_s^{-1}(p)\right) - p\left(V_s^{-1}(p) - y\right)] - d(y). \tag{3}$$

The FOC of (3), $d_y(y) = \delta p$, allows us to obtain the first best effort in the case the consumer invested k in the previous period:

$$y^* = d_y^{-1}(\delta p). \tag{4}$$

The marginal benefit of exerting effort is represented by the present value of the savings obtained through such an effort. Such a value is obtained by multiplying the quantity (i.e., 1) saved by one unit of effort by its present value δp. In equilibrium, the marginal benefit of the effort, δp, is thus equal to its marginal cost $d_y(y)$.

It should be noted, from (4), that the optimal level of effort increases in δp, i.e. the price works as an incentive for the consumer to increase her effort. In fact, the effort is a substitute for consumption. Therefore, when the price of consumption increases, it is optimal to increase the quantity-saving effort.

In $t = 0$, the consumer only invests k if the present value of the net surplus obtained from participation is greater than that obtained from no participation: $\delta^2 \left[V(s^*) - p\left(s^* - y^*\right)\right] - \delta d(y^*) - k \geq \delta^2 \left[V(s^*) - ps^*\right]$, i.e.

$$k \leq \epsilon^*, \tag{5}$$

where $\epsilon^* = \delta^2 py^* - \delta d(y^*)$ (or, using (4), $\epsilon^* = \delta^2 p d_y^{-1}(\delta p) - \delta d(d_y^{-1}(\delta p))$). Inequality (5) states that the commitment not to misbehave is only socially efficient if its benefit $\delta^2 py^*$, in terms of quantities saved, is higher than its direct cost k plus its indirect cost $\delta d(y^*)$ due to the first period effort.

From (5), the price affects the participation decision in two ways. First, a higher price increases the nominal value of the savings; second, since the savings are more valuable, the consumer is encouraged to exert more effort (i.e. $y^*(p)$ increases with p).

The interpretation of ϵ^* emerges directly from inequality (5). It should be recalled that the consumer's participation cost k is distributed uniformly over the range $[0, K]$. Since the consumer only participates if $k \leq \epsilon^*$, then ϵ^* represents the degree to which the consumer is willing to commit not to misbehave, and it depends on the cost of participation. For our purposes, ϵ^* can alternatively be interpreted as the customer base, i.e. as the set of consumer types who decide to participate because they have a participation cost $k \leq \epsilon^*$.

In order to determine the social planner's optimal policy, we now turn to welfare. Welfare can be expressed as

$$W = \int_0^{\epsilon^*} \left[\delta^2 \left[V(s^*) - c\left(s^* - y^*\right)\right] - \delta d(y^*) - k\right] df(k) + \int_{\epsilon^*}^{K} \left[\delta^2 \left(V(s^*) - cs^*\right)\right] df(k), \tag{6}$$

where the first term is the welfare of the consumer types who participate in $t = 0$, while the second term is the welfare of the consumer types who do not participate. From the FOC of expression (6) w.r.t. p, we find that the first best price corresponds to the marginal cost, as expressed by the following Proposition.

Proposition 1 *With time-consistent consumers, welfare is maximized at price $p^* = c$.*

Proof. See Appendix 1. ■

Marginal cost pricing implies that the first best effort is $y^* = d_y^{-1}(\delta c)$ and the consumer only participates if her cost k is lower or equal to the value $\epsilon^* = \delta^2 c d_y^{-1}(\delta c) - \delta d(d_y^{-1}(\delta c))$.

The result of Proposition 1 represents our benchmark for the analysis of the following sections.

5 Time-inconsistent consumers

We now assume that the consumer has present-biased preferences, which the literature (starting from the seminal work of Laibson, 1997, and, more recently, from the work of O'Donoghue and Rabin 1999a, 1999b and Della Vigna and Malmendier, 2004) usually represents through the simple functional form $U^t(u_t, ..., u_T) = u_t + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u_\tau$, where u_τ is the utility in period τ, the parameter δ is the standard long-term discount factor and the parameter β can be interpreted as the coefficient of short-term discounting. A time-consistent consumer has $\beta = 1$, while $\beta < 1$ represents a time-inconsistent preference for immediate gratification, and thus denotes short-term impatience. Since $\beta < 1$, a myopic consumer excessively discounts future flows. This results in self-control problems and it originates a bias between what the consumer does and what she thinks she should do, in terms of her own long-term best interest.

In this section, we study the time-inconsistent consumer's decisions in terms of effort (Section 5.1) and, going backward, participation (Section 5.2), given a generic price p. We then use these results to define the welfare-maximizing price (Section 5.3), and we compare it with our benchmark for time-consistent consumers. We denote the results of this section, pertaining to time-inconsistent consumers, with the superscript *inc*.

5.1 Incentives for effort

In $t = 2$, the consumer chooses the level of consumption $x^{inc} = V_s^{-1}(p) - y$, thus achieving the level of service $s^{inc} = V_s^{-1}(p)$. Proceeding backward to the previous period, and provided that she participated by paying k in $t = 0$, the consumer chooses the effort y^{inc} in $t = 1$ in order to maximize $\beta\delta\left[V(V_s^{-1}(p)) - p(V_s^{-1}(p) - y)\right] - d(y)$, whose FOC, $d_y(y) = \beta\delta p$, allows the following effort to be obtained:

$$y^{inc} = y^{inc}(p, \beta) = d_y^{-1}(\beta\delta p). \tag{7}$$

Equation (7) shows that, in the consumer's optimum, the marginal cost of effort $d_y(y)$ is equal to the marginal benefit $\beta\delta p$. By comparing (7) with (4), it is easy to see that, given the same price p, and provided that the consumer participated in the previous period, the myopic consumer under-exerts effort in $t = 1$, i.e. $y^{inc} < y^*$ for any p. The reason for this is that the benefits of effort are accrued in the future, while its costs are immediate, therefore the benefits are underweighted by a present-biased consumer and the effort is lower than in the first best. Even when the consumer invests k in $t = 0$, she will under-perform in terms of effort in $t = 1$, thus causing excessive consumption, given the investment of the previous stage.

In order to induce a present-biased consumer to exert a higher level of effort in $t = 1$, it would be sufficient to raise the price. In fact, since $y^{inc} = d_y^{-1}(\beta\delta p)$ and $y^* = d_y^{-1}(\delta c)$, it would be sufficient to impose a price $p = c/\beta$ to achieve the first best level of effort. The price here works as an incentive by increasing the marginal benefit of effort, i.e. the value of the quantity saved for each additional unit of effort[13].

However, it results that the rise in price does not come without its own cost, and the reason for this should be ascribed to the source of the distortion. If the consumer's bias involves misbehaviour rather than misconsumption, the correction of the distortion of y through the price necessarily entails some welfare loss.

[13]The fact that self-control effort and consumption are substitutes does not have an impact on our conclusions, but only on the direction of incentives (rewards instead of penalties). If, for example, the utility is expressed by the function $V(\min\{x, y\})$, the first best level of self-control effort can be achieved by imposing a price equal to $p = c - \frac{d_y}{\delta}(1/\beta - 1)$, which is lower than the marginal cost when $\beta < 1$.

When self-control problems affect consumers to the same degree (i.e., the population of consumer types is homogeneous in β), raising the price above the marginal (social) cost can completely correct the over-consumption problem (O'Donoghue and Rabin, 2006). This result is well known in the literature on sin taxes pertaining to myopic consumption.

However, accounting for consumers' behaviour dramatically affects the welfare effects of the policy. In fact, if the consumer's bias only concerns a consumption decision, the price is merely a market instrument which can achieve the first best if all the consumers display the same degree of myopia (i.e. the same β)[14]. Conversely, if we account for the possibility of consumers exerting self-restraint, by means, for example, of a quantity-saving, non-marketed effort, the price is not only a market instrument (through which the consumer decides on the quantity to consume), it is also an incentive for good behaviour. In this case, imposing a price $p = c/\beta$ above the marginal cost solves the misbehaviour problem, and makes the consumer choose the first best level y^* of effort, but it will later cause underconsumption of x *given* y, thus causing a welfare loss, as $V(s) = V(V_s^{-1}(p))$. As a consequence, raising the price has negative side-effects on the consumers' surplus, *even when all the consumers display myopia to the same degree*. Therefore, a trade-off emerges between the welfare gain of improving the consumers' behaviour and the welfare loss due to the higher price, even when consumers are homogeneous in terms of the time-inconsistency parameter β.

5.2 Incentives for participation

We now analyse the consumer's choice in $t = 0$. As discussed by Strotz (1955) and by Phelps and Pollak (1968), the behaviour of consumers with time-inconsistent preferences depends on their beliefs about their own future behaviour. Two polar cases have been used in the literature: *sophisticated* agents, who have rational expectations about their future selves and therefore are fully aware of their self-control problems and correctly predict their future behaviour, and *naive* agents (see also O'Donoghue and Rabin, 2001), who do not recognise that they cannot make consistent plans over time and therefore believe their future selves

[14]In fact, the concern expressed in the literature on optimal sin taxes is generally not about when all the consumers are equally myopic, but when they are heterogeneous in their myopia, so that helping irrational consumers is detrimental to rational ones. See, e.g., O'Donoghue and Rabin (2003, 2006) and Haavio and Kotakorpi (2011).

will behave exactly according to their long-term preferences. The consumers' unawareness is motivated by the experimental evidence on overconfidence about positive personal attributes (Larwood and Whittaker, 1977; Svenson, 1981) and is consistent with field evidence on investment (Madrian and Shea, 2001), task completion (Ariely and Wertenbroch, 2002), and health club attendance (Della Vigna and Malmendier, 2004).

Given that sophisticated and naive consumers differ, as far as their expectations about their future behaviour are concerned, we denote with $\hat{\beta} \in [\beta, 1]$ the consumer's expectation in $t = 0$ about the discount factor β that her future self will apply in $t = 1$. The higher $\hat{\beta}$ is, the higher the consumers' degree of naivety. In fact, a sophisticated consumer is more aware of her time-inconsistency problem than a more naive consumer and she anticipates her future behaviour more precisely. Therefore, her short-run discount factor $\hat{\beta}$ is closer to β than the $\hat{\beta}$ of a more naive consumer[15].

The belief $\hat{\beta}$ regarding one's own myopia plays a crucial role in the consumer's decision in $t = 0$ about whether to participate or not. In fact, she will only pay k if the savings are high enough, i.e. only if she expects to exert sufficient effort in the future to cover the entry cost k.

In $t = 0$, the consumer expects that her effort will be $y_0^{inc} = y_0^{inc}(p, \hat{\beta}) = d_y^{-1}(\hat{\beta}\delta p)$ in the next period. It should be noted that a higher price improves consumer behaviour in $t = 1$ as she will raise her effort. It should also be noted that $y_0^{inc}(p, \hat{\beta}) \neq y^{inc}(p, \beta)$: the former is the effort which, in $t = 0$, the consumer believes she will exert in $t = 1$, while the latter is the effort that she will actually exert[16].

Given a generic price p, the consumer invests k in $t = 0$ only if

$$\beta \left[\delta^2 \left(V(s^{inc}) - p\left(s^{inc} - y_0^{inc}\right)\right) - \delta d(y_0^{inc})\right] - k \geq \beta \left[\delta^2 \left(V(s^{inc}) - ps^{inc}\right)\right]. \tag{8}$$

Through straightforward simplifications, condition (8) can be rewritten as $k \leq \epsilon^{inc}(p, \beta, \hat{\beta})$, where $\epsilon^{inc}(p, \beta, \hat{\beta}) = \beta\left(\delta^2 p y_0^{inc} - \delta d(y_0^{inc})\right)$.

[15]In Section 7.2, we perform a numerical analysis in which we consider the two extreme cases of a fully sophisticated consumer, with $\hat{\beta} = \beta$, and a fully naive one, with $\hat{\beta} = 1$.

[16]In order to simplify the notation, from now on we will not report the full functional forms for $y_0^{inc}(\cdot)$ and $y^{inc}(\cdot)$.

It is worth analysing the effect of the price on ϵ^{inc}. As $d_y(y_0^{inc}) = \hat{\beta}\delta p$ from the FOC, we find that

$$\frac{\partial \epsilon^{inc}}{\partial p} = \beta \left(\delta^2 y_0^{inc} + \delta^2 p \frac{\partial y_0^{inc}}{\partial p}(1-\hat{\beta}) \right) > 0. \tag{9}$$

A higher price increases the probability of a consumer participating, by shifting the position $\epsilon^{inc}(\cdot)$ of the type of the marginal consumer, i.e. the one whose cost k makes her indifferent about participating or not.

In this framework, the benefits of raising the price are twofold. First, the price has a direct positive effect on the effort y^{inc}, which descends from the fact that the price determines the value py of the saved quantity. Second, the price has a positive indirect effect on the consumer's participation in $t = 0$, which descends from the fact that the higher expected effort y_0^{inc} makes it worth sinking the entry cost k.

5.3 The optimal price

Given a price p, welfare is given by

$$W = \int_0^{\epsilon^{inc}} \left[\delta^2 \left(V(s^{inc}) - c\left(s^{inc} - y^{inc}\right)\right) - \delta d(y^{inc}) - k\right] df(k) + \int_{\epsilon^{inc}}^{K} \left[\delta^2 \left(V(s^{inc}) - cs^{inc}\right)\right] df(k). \tag{10}$$

A quick glance at the welfare function in (10) suggests that the incentives on behaviour in the short-term (aimed at increasing y^{inc}, with $y^{inc} = y^{inc}(p, \beta)$) and long-term (aimed at increasing ϵ^{inc}, with $\epsilon^{inc} = \epsilon^{inc}(p, \beta, \hat{\beta})$) interact. Intuitively, a person can only exert effort in the short-term if they have decided to participate in the long-term; moreover, the social benefit of participating in the long-term depends on the self-control effort one exerts later on. Therefore, the optimal price needs to account for the intertemporal effect created by each type of incentive.

Before exploring this interaction, we characterise the optimal price.

Proposition 2 *A solution to the welfare maximization problem exists in the case of a time-inconsistent consumer. Moreover, it entails $p^{inc} \geq c$ for all $\beta \in [0, 1]$.*

Proof. See Appendix 1. ■

On the basis of Proposition 2, it is possible to state that when a consumer is myopic, a higher price than the marginal (social) cost helps to correct the misbehaviour by raising the benefits of good behaviour, i.e. by raising the value of the savings allowed by it. A higher price than the marginal social cost is needed to both induce effort, and to provide the incentive to participate.

In order to better understand the forces at play when determining the level of the optimal price incentives, let us express the FOC of the welfare function in (10) w.r.t. p. The FOC is reported in (11), where, for convenience, we denote the second and third terms with Λ_1 and Λ_2:

$$\underbrace{\delta^2 \frac{p-c}{V_{ss}(s^{inc})}}_{DWL} + \frac{1}{K}\left(\underbrace{\epsilon^{inc}\left(\delta^2 c - \delta d_y(y^{inc})\right)\frac{\partial y^{inc}}{\partial p}}_{\Lambda_1} + \underbrace{\left(\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}\right)\frac{\partial \epsilon^{inc}}{\partial p}}_{\Lambda_2} \right) = 0. \quad (11)$$

Condition (11) presents the marginal social benefits and the marginal social cost of raising the price above c. When $p^{inc} > c$, the first term, $\delta^2 \frac{p-c}{V_{ss}}$, is negative, due to $V_{ss} < 0$; it represents the marginal social cost caused by the price increase (i.e., the deadweight loss). In fact, a price $p^{inc} > c$ lowers the consumer's surplus, due to a reduction in the level of service by $1/V_{ss}$; such a cost occurs in $t = 2$ and is thus discounted by δ^2.

The second and third terms in (11) represent the marginal benefits of increasing the price that stem from the higher effort and participation, respectively. Such components are discussed hereafter.

The benefit of a higher effort (Λ_1). The first component, $\Lambda_1 = \epsilon^{inc}\left(\delta^2 c - \delta d_y(y^{inc})\right)\frac{\partial y^{inc}}{\partial p}$, constitutes the marginal social benefit of a price increase due to an increase in effort. From the expression of Λ_1, this depends on: i) the marginal value of effort (expressed by the factor $\epsilon^{inc}\left(\delta^2 c - \delta d_y(y^{inc})\right)$) and ii) the extent to which the effort increases due to the higher price (i.e., $\frac{\partial y^{inc}}{\partial p}$). It should be noted that the marginal value of effort accounts for both the marginal value of effort for the individual consumer, conditional to her participation (namely $(\delta^2 c - \delta d_y(y^{inc}))$), and for the ex-ante probability of participating (namely, ϵ^{inc}), as the value of effort can only be appropriated if the consumer participated in $t = 0$, which occurs for the ϵ^{inc} range of different consumer types.

A higher price leads the current base ϵ^{inc} of consumer types who participated in stage 0 to increase their effort in $t = 1$ by $\frac{\partial y^{inc}}{\partial p}$, thus gaining the amount $\delta^2 c - \delta d_y(y^{inc})$ from each additional unit of effort.

The benefit of a higher participation (Λ_2). Analogously to the benefit of effort, the social benefit of increasing participation depends on i) the marginal value of participation ($\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$) and ii) on how much participation increases due to the price (namely, $\frac{\partial \epsilon^{inc}}{\partial p}$).

The $\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$ term in (11) concerns the value of participation of the marginal consumer, i.e. of the consumer type $k = \epsilon^{inc}$ who only decides to participate after the price increase.

The $\frac{\partial \epsilon^{inc}}{\partial p}$ term in condition (11) is related to the effect of the price on the likelihood of a consumer participating. A higher price shifts the position of the marginal consumer toward the right by $\frac{\partial \epsilon^{inc}}{\partial p}$, thus increasing the consumer's participation: the price works as an incentive to participate.

Overall, the marginal social benefit of a price increase, $\Lambda_1 + \Lambda_2$, is given by the value generated by the greater effort for the current base of consumers, multiplied by the extent to which effort increases as a result of the incentive, plus the value of inducing new consumers to participate (and hence exert effort), multiplied by the 'number'of new consumer types who decide to participate.

The following section addresses the effect of consumer's myopia on these two components, in order to understand how the optimal price depends on the level of the consumer's present-bias.

6 Effect of consumer's myopia on optimal incentives

As $s^{inc} = V_s^{-1}(p)$ does not depend on β, the deadweight loss (the term DWL in (11)) is constant, regardless of the level of consumer's myopia, given the price. It follows that, in order to understand the role β plays in the optimal price, it is necessary to look at the other terms, $\Lambda_1 + \Lambda_2$, which are functions of β through $y^{inc}(p, \beta)$ and $\epsilon^{inc}(p, \beta, \hat{\beta})$.

The degree of myopia operates in opposite directions on the components of Λ_1 and Λ_2.

First, both $\frac{\partial y^{inc}}{\partial p} = \frac{\beta\delta}{d_{yy}}$ and $\frac{\partial \epsilon^{inc}}{\partial p}$ (from (9)) increase in β: the higher the consumer's myopia is, the less effective a price increase in inducing behavioural change. Myopic consumers are more hesitant about responding to incentives, as they can only be rewarded in the future, (when the benefits of good behaviour materialise), and thus discount them excessively.

Second, myopia has direct effects on the marginal values of self-control effort and participation through the levels of y^{inc} and ϵ^{inc}, respectively. In fact, the marginal value of self-control effort, i.e. the factor $\epsilon^{inc}(\delta^2 c - \delta d_y(y^{inc}))$ in Λ_1, decreases with y^{inc}, which in turn is low for high myopia. Symmetrically, the marginal value of participation, i.e. $\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$, decreases with ϵ^{inc}, which is low for high myopia. The problems of under-exertion of effort and under-participation are more severe when the degree of myopia is high, thus causing the social inefficiency to become worse.

Third, intertemporal effects are present, so that myopia affects the marginal value of effort through participation and, viceversa, the marginal value of participation through effort. In fact, myopia decreases the share ϵ^{inc} of consumer types affected by a policy that targets the self-control effort. This negatively affects the marginal value of self-control effort, i.e. $\epsilon^{inc}(\delta^2 c - \delta d_y(y^{inc}))$. When myopia is high, a policy that targets the effects of misbehaviour in the short-term (self-control effort) is partially thwarted by the fact that it only works on the small subset of consumers who participate in the long-term. Intuitively, with high myopia, incentives are partially wasted by the limited scope of the campaign, as they can only raise the effort exerted by the few people who joined, for example, a wellness program. Symmetrically, myopia decreases the effort y^{inc} exerted by consumers who participated. This negatively affects the marginal value of participation, i.e. $\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$. Intuitively, in the presence of a strong present bias, policies aimed at significantly increasing the number of people who join a wellness program are partially thwarted by the fact that these individuals will rarely attend.

The previous analysis suggests that the optimal level of incentives is non-monotonic with respect to the degree of myopia. The following Corollaries formalise this result and show that the optimal price distortion shows a maximum for internal levels of β, while it is zero for the two extreme cases of $\beta = 1$ (fully rational) and $\beta = 0$ (fully myopic) consumers.

Corollary 1 *The optimal price is* $p^{inc} = c$ *for* $\beta = 1$ *or* $\beta = 0$*, and* $p^{inc} > c$ *for all* $\beta \in (0, 1)$*.*

Corollary 2 *The optimal price is maximized for* $\beta \in (0, 1)$*.*

Proof. See Appendix 1. ■

As a result of the intertemporal effects, the social value of policies that address effort is undermined by low participation rates, and the low levels of effort symmetrically undermine the convenience of policies that address participation. This direct relationship between participation and effort that emerges for high levels of myopia is actually not surprising, on account of the complementarity between effort and participation decisions. On the other hand, when we account for the second effect, the opposite is true. For sufficiently low levels of myopia, incentives decrease the marginal value of effort for each individual consumer, and the impact is greater due to the greater participation. Hence, the social value of implementing the policy is low, despite the fact that both effort and participation increase.

From another perspective, the interaction between the short- and long-term behaviour dissipates the social benefits of incentives that target it, both when the present bias is sufficiently strong and when it is weak, and, interestingly, this happens *despite* the complementarity between efforts. This dissipation makes such a policy unworthwhile, regardless of how difficult it is to induce consumers to participate. [17]

The next section delves more deeply into the impact of the consumer's awareness on the incentives on behaviour and participation.

[17] It should be noted that myopia is necessary for the non-monotonicity result, as it is responsible for the trade-off between benefits and effectiveness of incentives. Conversely, in the case of a distortion caused by an externality, the optimal incentive would always increase with the level of the distortion. This result can be obtained immediately by modelling a welfare function that differs from the consumer's objective function as a result of the introduction of an externality term as a function of the effort y. For example, assuming a fully rational consumer ($\beta = 1$), the welfare function could be expressed by adding the externality term $\psi(y)$ in (6).

7 Effect of consumer's awareness on incentives

In a similar way to the previous analysis, in this section we study how a change in consumer's awareness $\hat{\beta}$ affects the terms in (11), through $\epsilon^{inc}(p, \beta, \hat{\beta})$.

The benefit of a higher effort (Λ_1). The extent of this benefit depends on the degree of naivety of the consumer. While the effect of price on effort $\frac{\partial y^{inc}}{\partial p}$ does not depend on $\hat{\beta}$ (i.e., in $t = 1$ we will increase our effort to the same extent regardless of our naivety), the value of marginal effort $\epsilon^{inc}\left(\delta^2 c - \delta d_y(y^{inc})\right)$ does, per the size of the customer base ϵ^{inc}. In fact, since

$$\frac{\partial \epsilon^{inc}}{\partial \hat{\beta}} = \frac{\beta \delta^3 p^2}{d_{yy}(\hat{\beta}\delta p)}(1 - \hat{\beta}) > 0, \tag{12}$$

a higher degree of naivety increases the probability of a consumer participating in stage 0.

Condition (12) shows that the more one (naively) expects to behave rationally in the future, the less severe the participation problem is at earlier stages, as a naive consumer overestimates the benefits of her future behaviour[18]. A naive consumer is more likely to participate than a sophisticated one, so that the benefits from the additional effort (which are the same, regardless of a person being naive or sophisticated) are appropriated with a higher probability.

It follows that Λ_1 is higher for a naive consumer than for a sophisticated one, and this implies a higher optimal price the higher $\hat{\beta}$ is.

The benefit of higher participation (Λ_2). Consumer's awareness affects Λ_2 through ϵ^{inc}, by influencing both the marginal effect $\frac{\partial \epsilon^{inc}}{\partial p}$ of the price on participation, and the marginal value $\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$ of participation.

As far as the marginal value of participation is concerned, it should be recalled that $\frac{\partial \epsilon^{inc}}{\partial \hat{\beta}} > 0$: the more sophisticated a consumer is, the lower ϵ^{inc}, i.e. the marginal consumer has a lower entry cost k, and the actual social benefit from inducing her to participate is

[18] In other terms, if we naively overestimate our willpower to exert effort, we are more likely to sink the cost of joining a gym, applying for a master program or retrofitting a house, even though later we will underperform.

higher. As a consequence, the marginal value of participation is higher for a sophisticated consumer than for a naive one. Thus, shifting the marginal consumer provides a higher social value when the consumer is sophisticated rather than naive.

The marginal effect of $\hat{\beta}$ on participation is ambiguous. In fact, the value of

$$\frac{\partial}{\partial \hat{\beta}} \left(\frac{\partial \epsilon^{inc}}{\partial p} \right) = \frac{\beta \delta^3 p}{d_{yy}(\hat{\beta}\delta p)} \left(1 - \hat{\beta}\right) \left(2 - \hat{\beta}\delta p \frac{d_{yyy}(\hat{\beta}\delta p)}{(d_{yy}(\hat{\beta}\delta p))^2}\right) \tag{13}$$

can be positive or negative, depending on the curvature of the effort cost function. Intuitively, a higher price increases the expected effort y_0^{inc}, and this leads to both benefits and costs. On the one hand, the expected savings py_0^{inc} are higher (both because of the higher price and the higher effort). This effect is unambiguously stronger for naive consumers as they have a higher y_0^{inc}. On the other hand, a greater effort implies a higher effort cost. If these effort costs increase rapidly (d_{yyy} is large), naive consumers, who expect to exert more effort than sophisticated ones, achieve a lower net expected benefit from the effort. This means that, when the price increases, they do in fact increase their participation, but they might do so more 'slowly' than more sophisticated consumers. Conversely, if the effort costs increase relatively slowly (i.e., d_{yyy} is small enough), the value of (13) is positive, which means that the increase in $\hat{\beta}$ increases the rate $\partial \epsilon^{inc}/\partial p$ with which incentives increase participation. If the effort costs are not too steep, an increase in the price is more effective in inducing the consumer to participate, the higher the consumers' naivety is.

7.1 A synthesis

The consumer's degree of awareness of her own myopia does not affect her actual short-term behaviour: in fact, the effort y^{inc} is the same, regardless of $\hat{\beta}$, and so is the extent to which y^{inc} increases because of the incentive. However, the degree of awareness does affect a consumer's earlier expectation about effort, with consequences on the participation decision. In fact, when consumers are naive, they overestimate their future willingness to engage in quantity-saving activities. Therefore, they are more likely to sink the participation costs: the under-participation problem is less severe for naive consumers than for sophisticated ones.

This has both positive and negative effects on the optimal level of incentives.

On the one hand, the greater willingness of naive consumers to participate means that the benefits of improved self-control can be attained by a larger customer base. This implies that the effects of incentives on self-control effort are socially more valuable when consumers are naive rather than sophisticated.

On the other hand, the greater willingness of naive consumers to participate means that they have a lower marginal value of participation, as the problem of under participation is less severe. This implies that the effects of incentives on participation are socially more valuable when consumers are sophisticated rather than naive.

Finally, a third effect, which can go both ways, is related to the effectiveness of the price on raising participation. If the effort costs do not increase too rapidly, a unit rise in the price increases the probability of the consumer participating to a larger extent the more naive the consumer is. Conversely, if the effort costs increase sharply, the price is more effective in convincing sophisticated consumers to participate, as they anticipate the need to exert less effort than naive ones, and thus expect to have much lower effort costs (i.e., a greater benefit).

The first notable policy implication from the first and second effects is that the marginal benefit of incentives, in terms of effort, related to the benefit in terms of participation, is larger for naive consumers than for sophisticated ones. In fact, y^{inc} is the same for both types, but naive consumers show a higher ϵ^{inc}, which increases the social value of incentivising effort (through a greater outreach of the policy) and decreases the value of incentivising participation (because of the larger participation costs for the additional consumers).

A second policy implication is that, even if we ignore the ambiguous effects of awareness on the effectiveness of incentives, incentive policies on effort have a wider outreach for more naive consumers, but a smaller value for each consumer. The presence of these countervailing effects suggests that the optimal price might not be monotonic in the degree of consumers' naivety. The following proposition formalises this result and provides the condition under which one of the effects prevails over the others.

Proposition 3 *If myopia is sufficiently high (low), the optimal price is higher (lower) for* *naive consumers than for sophisticated ones.*

Proof. See Appendix 1. ■

Proposition 3 states that when a consumer has a strong present bias, the optimal price incentive is higher for a naive consumer than for a sophisticated one. Conversely, when the consumer's myopia is low, sophisticated consumers should be given a higher incentive than naive ones.

For high levels of myopia, the benefit of incentives on effort is significantly higher for naive consumers than for sophisticated ones, on account of the wider outreach of the policy.

Conversely, for low levels of myopia, the benefit of incentives on effort is similar for naive and for sophisticated consumers, but incentives offer sophisticated consumers a much higher benefit in terms of participation than naive ones. Because of the notable participation-related effect, prices are higher for sophisticated consumers.

7.2 An illustrative example

In order to gain a better insight into the implications of consumer's awareness on the optimal price, a numerical and graphical analysis has been conducted.

To this aim, let us assume $d(y) = y^2$. Therefore, in $t = 1$, the consumer exerts the effort $y^{inc} = 2\beta\delta p$ from (7), but in $t = 0$ she expects that she will exert $y_0^{inc} = 2\hat{\beta}\delta p$. The marginal cost of production of x is assumed constant and equal to 10.

Figure 1 shows the optimal price incentive (on the vertical axis) as a function of the level of myopia β. The two curves represent the optimal price for two levels of consumer's awareness: a completely naive consumer ($\hat{\beta} = 1$) and a completely sophisticated one ($\hat{\beta} = \beta$).

In panels (a) and (b) in Figure 1, we assume a utility function with constant elasticity (equal to -10 and -1, respectively). In panels (c) and (d) in Figure 1, we assume a radical – panel (c) – and an exponential – panel (d) – $V(s)$ curve.

The numerical analysis confirms that the optimal price is always greater than the marginal cost, it is maximized for $\beta \in (0, 1)$, and it converges to the marginal cost both when the consumer is very myopic (β goes to 0) and when she is nearly rational (β goes to 1), in line with the results of Proposition 2 and Corollaries 1 and 2. Moreover, when the consumer's

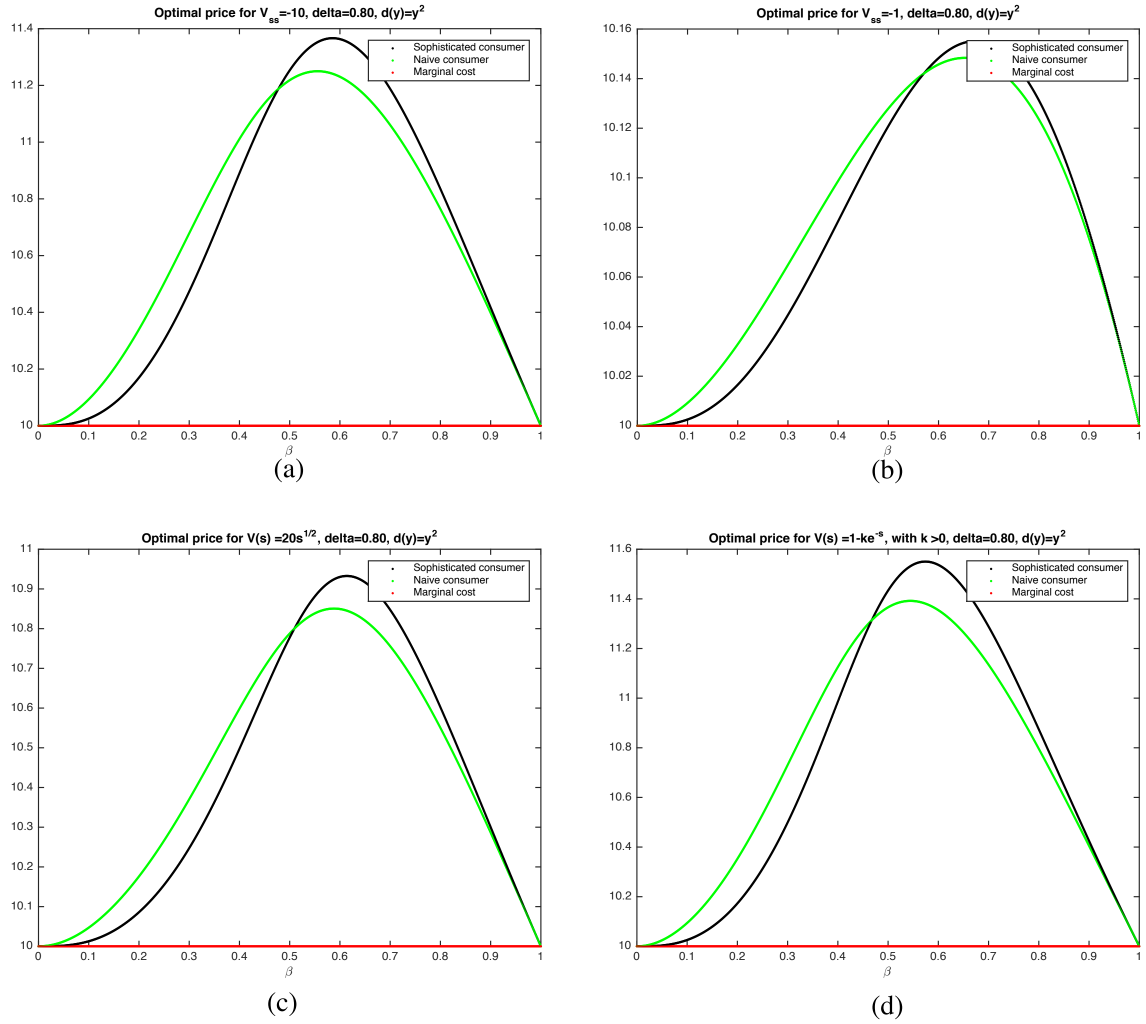

Figure 1: Optimal price for a naive and sophisticated consumer, quadratic utility functions with constant second derivatives ((a) and (b)), radical (c) and exponential (d) utility functions. Marginal costs equal to 10.

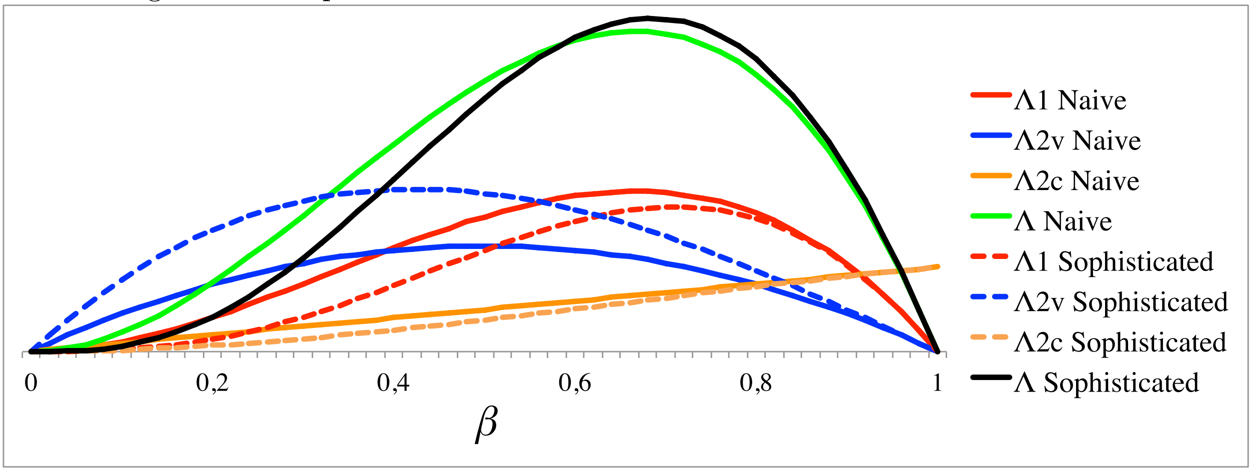

Figure 2: Marginal benefits of a price increase, for a naive and a sophisticated consumer, where Λ_2 is decomposed into $\Lambda_{2v} = \delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$ and $\Lambda_{2c} = \frac{\partial \epsilon^{inc}}{\partial p}$.

myopia is sufficiently high, the optimal price is higher when the consumer is naive. Conversely, when the myopia is low, the optimal price is higher for sophisticated consumers than for naive ones, consistently with the results of Proposition 3.

For a better intuition of the above result, it is convenient to look at Figure 2, where the marginal benefits of increasing the price, in terms of effort and participation, Λ_1 and Λ_2, are represented as functions of β on the horizontal axis.

In the Figure, $\Lambda_2 = \Lambda_{2v} \cdot \Lambda_{2c}$ has been disentangled into its two components, where $\Lambda_{2v} = \delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc}$ is the value of participation and $\Lambda_{2c} = \frac{\partial \epsilon^{inc}}{\partial p}$ is the effect of the price on the change in participation. The solid lines refer to a completely naive consumer ($\hat{\beta} = 1$), while the dashed lines refer to a completely sophisticated one ($\hat{\beta} = \beta$). It should first be noted that the benefit of incentives, in terms of effort, relative to the benefit in terms of participation, is higher for naive consumers than for sophisticated ones. The much wider outreach of the policy for high levels of myopia (i.e., low β) implies that the effort effect is dominant, so that the price is higher for naive consumers. Conversely, for low levels of myopia (i.e., high β), the advantage of sophisticated consumers, in terms of participation, overrides the advantage of naive consumers in terms of effort, so that the price is higher for sophisticated consumers.

8 Conclusion

The literature on consumer's present bias is generally concerned with the effects of people's myopic decisions on consumption. We here contribute to this literature by modelling myopic behaviour decisions that involve the distortion of an effort, rather than of the demand of goods. This analysis provides an interpretation of the empirical puzzle about the limited effectiveness of financial incentives in the long term.

We show that a large incentive provided to correct the severe misbehaviour of a consumer might be suboptimal if the degree of myopia and awareness of the individual are not taken into account. In other words, we find that financial incentives should be larger for intermediate levels of myopia, while they are negligible for both very high and very low levels of myopia.

The non-monotonic profile of the socially optimal level of financial incentives arises from the trade-off between the value of incentives and their effectiveness. A very myopic consumer obtains a larger benefit from correcting her behaviour, but is also more difficult to convince, because she myopically disregards the future benefits of effort. Conversely, a consumer with very low myopia obtains limited advantages from improving her behaviour, but incentives are more effective because she recognises their importance.

Moreover, in a long-term perspective, this trade-off is complicated by the interplay of the levels of effort exerted at different stages. In particular, the myopic decision to underexert efforts dissipates the benefits of inducing program uptake, and, symmetrically, the low rates of program uptake of myopic consumers dissipate the benefits of inducing regular attendance. Interestingly, when the consumer's myopia is low, an increase in effort in later periods negatively influences the value of incentives on behaviour at earlier stages, such as participation decisions.

The consumers' awareness of their own myopia also influences the size of financial incentives in two opposite directions. One the one hand, incentive policies show a smaller outreach in the case of sophisticated consumers. In fact, sophisticated consumers anticipate that they will misbehave in the future, so that they have little interest in participating. On the other hand, the misbehaviour problem is more severe for sophisticated consumers, so that improving their behaviour is more valuable from a social point of view.

Incentives are being employed extensively by policymakers in order to encourage people to initiate healthy habits and continue them on a long-term basis. As recent as October 2017, a heated debate arose from the decision of an NHS commission in the UK to restrict surgery to obese and smokers, unless they lost weight and stopped smoking. However, empirical evidence has shown that such incentives do not seem to have produced a notable or lasting behavioural change. This paper provides useful insights into the causes of the disappointing results of such policies, and offers direction to policymakers for the design of optimal incentive schemes. Our results show that, due to the multifaceted implications of consumers' myopia, a policy that simply keeps increasing the price of misbehaviour might be too naive, socially costly and, more importantly, ineffective in inducing behavioural change.

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Appendix 1: Proof

Proof of Proposition 1. By substituting (1) and (4) in (6), the latter can be rewritten as

$$W = \delta^2 \left[V(V_s^{-1}(p)) - cV_s^{-1}(p)\right] + \frac{1}{K}\left(\delta^2 c d_y^{-1}(\delta p)\epsilon^* - \delta d(d_y^{-1}(\delta p))\epsilon^* - \frac{(\epsilon^*)^2}{2}\right).$$

In order to find the optimal price, we find the FOC of W w.r.t. p:

$$\delta^2 \frac{V_s(s^*) - c}{V_{ss}(s^*)} + \frac{1}{K}\left[\delta^2 c\left(\frac{\delta}{d_{yy}(y^*)}\epsilon^* + d_y^{-1}(\delta p)\frac{\partial \epsilon^*}{\partial p}\right) - \delta\left(\frac{\delta d_y(y^*)}{d_{yy}(y^*)}\epsilon^* + d(d_y^{-1}(\delta p))\frac{\partial \epsilon^*}{\partial p}\right) - \epsilon^*\frac{\partial \epsilon^*}{\partial p}\right] = 0.$$

Given that $V_s(s^*) = p$ and $d_y(y^*) = \delta p$, the previous expression becomes

$$\delta^2 \frac{p-c}{V_{ss}(s^*)} - \frac{1}{K}\frac{\delta^3 \epsilon^*}{d_{yy}(y^*)}(p-c) + \frac{1}{K}\frac{\partial \epsilon^*}{\partial p}\left(\delta^2 c d_y^{-1}(\delta p) - \delta d(d_y^{-1}(\delta p)) - \epsilon^*\right) = 0.$$

Since $\epsilon^* = \delta^2 p d_y^{-1}(\delta p) - \delta d(d_y^{-1}(\delta p))$, the expression becomes

$$\delta^2 \frac{p-c}{V_{ss}(s^*)} - \frac{1}{K}\frac{\delta^3 \epsilon^*}{d_{yy}(y^*)}(p-c) - \frac{1}{K}\frac{\partial \epsilon^*}{\partial p}\delta^2 d_y^{-1}(\delta p)\,(p-c) = 0,$$

which is verified for $p = c$. ■

Proof of Proposition 2 and Corollaries 1 and 2. We prove the Proposition and Corollaries through two Lemmas, concerning the characterisation (Lemma 1) and the existence (Lemma 2) of a solution to the social planner's problem, respectively.

Lemma 1 *If a solution exists, it entails $p^{inc} = c$ for $\beta = 1$ or $\beta = 0$, and $p^{inc} > c$ for all $\beta \in (0,1)$.*

Proof. Expression (10) can be rewritten as

$$W = \delta^2\left[V(V_s^{-1}(p)) - cV_s^{-1}(p)\right] + \frac{1}{K}\left(\delta^2 c d_y^{-1}(\beta\delta p)\epsilon^{inc} - \delta d(d_y^{-1}(\beta\delta p))\epsilon^{inc} - \frac{(\epsilon^{inc})^2}{2}\right).$$

In order to find the optimal price, we find the FOC of W w.r.t. p:

$$\delta^2 \frac{p-c}{V_{ss}(s^{inc})} + \frac{\beta\delta^3\epsilon^{inc}}{d_{yy}(y^{inc})}\frac{c-\beta p}{K} + \frac{\partial \epsilon^{inc}}{\partial p}\frac{\delta^2 c d_y^{-1}(\beta\delta p) - \delta d(d_y^{-1}(\beta\delta p)) - \epsilon^{inc}}{K} = 0, \qquad (14)$$

where $V_s(s^{inc}) = p$ and $d_y(y^{inc}) = \beta\delta p$. The proof proceeds in two steps. We first prove that $p^{inc} = c$ for $\beta = 0$ or $\beta = 1$. Secondly, we prove that $p^{inc} \leq c$ cannot be a solution to the social planner's problem for $\beta \in (0,1)$.

When $\beta = 0$, $\epsilon^{inc} = \frac{\partial \epsilon^{inc}}{\partial p} = 0$. Therefore, (14) becomes $K\delta^2 \frac{p-c}{V_{ss}(s^{inc})} = 0$, which is verified for $p^{inc} = c$. When $\beta = 1$, we also have $\hat{\beta} = 1$; therefore, from Proposition 1, $p^{inc} = c$.

Let us now prove that $p^{inc} > c$ for $\beta \in (0,1)$. Let us suppose, by contradiction, that $p^{inc} \leq c$. Thus, the sum of the first two terms of condition (14) would be strictly positive. Since $\frac{\epsilon^{inc}}{\partial p}$ is also positive, the welfare maximization problem has an internal solution when the last term, which we denote with $L(\beta, \hat{\beta})$, is strictly negative, i.e.

$$L(\beta, \hat{\beta}) = \delta c d_y^{-1}(\beta\delta p) - d(d_y^{-1}(\beta\delta p)) - \beta \left(\delta p d_y^{-1}(\hat{\beta}\delta p) - d(d_y^{-1}(\hat{\beta}\delta p))\right) < 0.$$

We now study the function $L(\beta, \hat{\beta})$ and show that, even when it is minimized, it is still positive for $p \leq c$, thus contradicting the hypothesis that $p^{inc} \leq c$ is an equilibrium when $\beta \in (0,1)$. To this aim, let us note that the function $L(\beta, \hat{\beta})$ always decreases in $\hat{\beta}$. This means that it is always mimimized when $\hat{\beta} = 1$, for all p and β. Moreover, when $\hat{\beta} = 1$, we obtain

$$\begin{aligned}
\frac{\partial L(\beta, 1)}{\partial \beta} &= \frac{\delta^2 cp}{d_{yy}(\beta\delta p)} - \frac{\beta\delta^2 p^2}{d_{yy}(\beta\delta p)} - \left(\delta p d_y^{-1}(\delta p) - d(d_y^{-1}(\delta p))\right) \\
\frac{\partial^2 L(\beta, 1)}{\partial \beta^2} &= \delta^2 p \frac{-p d_{yy}(\beta\delta p) - (c - \beta p)\delta p d_{yyy}(\beta\delta p)}{(d_{yy}(\beta\delta p))^2}.
\end{aligned}$$

Since from hypothesis $p \leq c$, then $c - \beta p > 0$; moreover, $d_{yy} > 0, d_{yyy} \geq 0$. Hence, $\frac{\partial^2 L}{\partial \beta^2} < 0$, which implies that $L(\beta, 1)$ is a strictly concave function in the support of β. As a consequence, it is minimized for either $\beta = 0$ or $\beta = 1$. In $\beta = 0$, $L(0, 1) = 0$. Conversely, in $\beta = 1$, $L(1, 1) = \delta(c - p)d_y^{-1}(\delta p)$, which is positive or null for $p \leq c$. It follows that when $p \leq c$, $\min L(\beta, \hat{\beta}) \geq 0$ for all $\beta \in (0,1)$. As a consequence, when $p \leq c$, $L(\beta, \hat{\beta}) \geq 0$ for all $\beta, \hat{\beta} \in (0,1) \times [\beta, 1)$. Therefore, the l.h.s. of (14) is strictly positive for $p \leq c$, which implies that there does not exist a solution with $p^{inc} \leq c$ for any $\beta \in (0,1)$. ■

Lemma 1 only shows that the optimal price must be higher than the marginal cost. However, in principle, the problem might not be closed. The following Lemma ensures that the problem is closed, and thus a solution always exists.

Lemma 2 *A solution to the welfare maximization problem always exists.*

Proof. When $\beta = 0$ or $\beta = 1$, a solution exists and it is equal to $p^{inc} = c$ from Lemma 1. Therefore, we only need to show that a solution exists when $\beta \in (0, 1)$.

First, let us observe that all the functions in (14) are continuous in p. Moreover, in $p = c$, the l.h.s. of (14) is strictly positive when $\beta \in (0, 1)$ by Lemma 1. Should the l.h.s. of (14) be strictly negative for some $p > c$ when $\beta \in (0, 1)$, then there exists some value of p such that the FOC in (14) is equal to zero by continuity, i.e a solution to the welfare maximization exists.

Let us consider the price $p = c/\beta$. The sum of the first two terms in (14) is strictly negative, while $\frac{\partial \epsilon^{inc}}{\partial p}$ is always positive. Let us focus on the third term, i.e. $L(\beta, \hat{\beta})$. We want to find the values of $\beta, \hat{\beta}$ so that $L(\beta, \hat{\beta})$ is maximized, and show that L is still negative for these values when p is sufficiently high. It should be noted that $L(\beta, \hat{\beta})$ decreases in $\hat{\beta}$. This implies that $L(\beta, \hat{\beta})$ is maximized when $\hat{\beta} = \beta$. By substituting $\hat{\beta} = \beta$ and $p = c/\beta$ in the expression of $L(\beta, \hat{\beta})$, the latter becomes

$$L(\beta, \beta) = \delta c d_y^{-1}(\delta c) - d(d_y^{-1}(\delta c)) - \beta \left(\delta \frac{c}{\beta} d_y^{-1}(\delta c) - d(d_y^{-1}(\delta c)) \right)$$

i.e. $L(\beta, \beta) = -(1 - \beta) d(d_y^{-1}(\delta c)) < 0$ for all $\beta \in (0, 1)$. Hence, the l.h.s. of (14) is strictly negative for $p = c/\beta > c$.

Since the l.h.s. of (14) is strictly positive for $p = c$, it is strictly negative for $p = c/\beta > c$ and it is continuous, there exists at least one price $p \in (c, c/\beta)$ so that the l.h.s. is equal to zero by continuity, i.e. a solution to the welfare maximization problem exists. ■ ■

Proof of Proposition 3. From the FOC condition, the term

$$\Lambda = \left(\frac{\delta^3 \beta c}{d_{yy}(y^{inc})} - \frac{\delta^3 \beta^2 p}{d_{yy}(y^{inc})} \right) \epsilon^{inc} + \left(\delta^2 c d_y^{-1}(\beta \delta p) - \delta d(d_y^{-1}(\beta \delta p)) - \epsilon^{inc} \right) \frac{\partial \epsilon^{inc}}{\partial p}$$

represents the marginal social benefit of increasing the price. Therefore, the higher the marginal benefit Λ is, the larger the optimal price distortion. Let us study function Λ:

$$\begin{aligned} \frac{\partial \Lambda}{\partial \beta} &= \left(\delta^3 c - 2\beta \delta^3 p \right) \frac{\epsilon^{inc}}{d_{yy}(y^{inc})} + \left(\delta^3 \beta c - \beta^2 \delta^3 p \right) \frac{\partial}{\partial \beta} \left(\frac{\epsilon^{inc}}{d_{yy}(y^{inc})} \right) + \\ &+ \left(\frac{\delta^3 p c}{d_{yy}(y^{inc})} - \frac{\beta \delta^3 p^2}{d_{yy}(y^{inc})} - \frac{\partial \epsilon^{inc}}{\partial p} \right) \frac{\partial \epsilon^{inc}}{\partial p} + \left(\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc} \right) \frac{\partial^2 \epsilon^{inc}}{\partial p \partial \beta} \end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 \Lambda}{\partial \beta^2} &= -2\delta^3 p \frac{\epsilon^{inc}}{d_{yy}(y^{inc})} + 2(\delta^3 c - 2\beta\delta^3 p)\frac{\partial}{\partial \beta}\left(\frac{\epsilon^{inc}}{d_{yy}(y^{inc})}\right) + (\delta^3 \beta c - \delta^3 \beta^2 p)\frac{\partial^2}{\partial \beta^2}\left(\frac{\epsilon^{inc}}{d_{yy}(y^{inc})}\right) + \\
&+ \left(-\delta^4 c p^2 \frac{d_{yyy}(y^{inc})}{(d_{yy}(y^{inc}))^3} - \delta^3 p^2 \frac{d_{yy}(y^{inc}) - \beta\delta p \frac{d_{yyy}(y^{inc})}{d_{yy}(y^{inc})}}{(d_{yy}(y^{inc}))^2} - \frac{\partial^2 \epsilon^{inc}}{\partial \beta^2}\right)\frac{\partial \epsilon^{inc}}{\partial p} + \\
&+ 2\left(\frac{\delta^3 p c}{d_{yy}(y^{inc})} - \frac{\delta^3 \beta p^2}{d_{yy}(y^{inc})} - \frac{\partial \epsilon^{inc}}{\partial \beta}\right)\frac{\partial^2 \epsilon^{inc}}{\partial p \partial \beta} + (\delta^2 c y^{inc} - \delta d(y^{inc}) - \epsilon^{inc})\frac{\partial^3 \epsilon^{inc}}{\partial p \partial^2 \beta},
\end{aligned}
$$

where $\frac{\partial}{\partial \beta}\left(\frac{\epsilon^{inc}}{d_{yy}(y^{inc})}\right) = \frac{\frac{\partial \epsilon^{inc}}{\partial \beta} d_{yy}(y^{inc}) - \delta p \epsilon^{inc} \frac{d_{yyy}(y^{inc})}{d_{yy}(y^{inc})}}{(d_{yy}(y^{inc}))^2}$. When β goes to zero, the values of function Λ and of its first derivative are also zero. However, the second derivative is positive. In fact,

$$
\left.\frac{\partial^2 \Lambda}{\partial \beta^2}\right|_{\beta=0} = 2\frac{\delta^3 c}{d_{yy}(y^{inc})}\frac{\partial \epsilon^{inc}}{\partial \beta} + 2\left(\frac{\delta^3 c p}{d_{yy}(y^{inc})} - \frac{\partial \epsilon^{inc}}{\partial \beta}\right)\frac{\partial^2 \epsilon^{inc}}{\partial p \partial \beta}. \tag{15}
$$

The first component of (15) is positive. The second component is positive, as the term in the parenthesis is positive. To see this, let us denote with $q(\beta) = \delta^2 c y^{inc} - \epsilon^{inc}$. We observe that $\frac{\partial q}{\partial \beta} = \frac{\delta^3 c p}{d_{yy}(y^{inc})} - \frac{\partial \epsilon^{inc}}{\partial \beta}$. The function $q(\beta)$ is always positive (as $q(\beta) > \delta^2 c y^{inc} - d(y^{inc}) - \epsilon^{inc} \geq 0$). Moreover, $\frac{\partial^2 q}{\partial \beta^2} = -\delta^4 c p^2 \frac{d_{yyy}(y^{inc})}{(d_{yy}(y^{inc}))^3} < 0$. Therefore, the function $q(\beta)$ increases in the interval $[0,1]$, i.e. its first derivative is positive. We conclude that function Λ increases in the neighbourhood of $\beta = 0$. We now prove that it increases faster for naive consumers than for sophisticated ones. The value of the second derivative of Λ in the neighbourhood of $\beta = 0$ is zero for a fully sophisticated consumer ($\hat{\beta} = 0$). The value of the second derivative of Λ in the neighbourhood of $\beta = 0$ is equal to $2\frac{\delta^3 c}{d_{yy}(y^{inc})}(\delta^2 p y_0^{inc} - \delta d(y_0^{inc})) + 2\left(\frac{\delta^3 c p}{d_{yy}(y^{inc})} - (\delta^2 p y_0^{inc} - \delta d(y_0^{inc}))\right)\delta^2 y_0^{inc} > 0$ for a fully naive consumer ($\hat{\beta} = 1$).

Let us now determine the sign of $\frac{\partial \Lambda}{\partial \hat{\beta}}$ for a sufficiently high β:

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial \hat{\beta}} &= \frac{\beta\delta^3 p(1-\hat{\beta})}{d_{yy}} \cdot \left[\frac{\beta\delta^3 p}{d_{yy}}\left(c - \beta p - p\hat{\beta}(1-\hat{\beta})\right) - \beta\delta^2 p d_y^{-1}(\hat{\beta}\delta p) + \right. \\
&\left. +2\left(\delta^2 c d_y^{-1}(\beta\delta p) - \delta d(d_y^{-1}(\beta\delta p)) + \beta\delta d(d_y^{-1}(\hat{\beta}\delta p)) - \beta\delta^2 p d_y^{-1}(\hat{\beta}\delta p)\right)\right].
\end{aligned}
$$

When β goes to 1, $\hat{\beta}$ also goes to 1, as $\hat{\beta} \geq \beta$. Therefore, $\left.\frac{\partial \Lambda}{\partial \hat{\beta}}\right|_{\beta=1} = 0^+ \cdot \left[-\delta^2 p d_y^{-1}(\delta p)\right] = 0^-$: the optimal price is lower for naive consumers than for sophisticated ones. ■