# Multidimensionality and Complexity in Scoring Rule Auctions:

## an Experimental Investigation

*Preliminary Version*

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22/09/2018 HSE Perm

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# Motivation & Aim

- Scoring rules auctions (SRA): awarding mechanism where the offer includes price and level(s) of quality, weighted by a scoring function. The resulting highest score wins.
- EU Directive 2014/24/EU strongly supports adoption of SRA (and similarly in other national contexts).
- In 2016, 72 % of all EU procurement auctions above 150.000 euro are SRAs.

# What we do and how we do

- Explore the SRA's higher theoretical efficiency and its lower observed performance
- Experimentally investigates bidding behaviour and related performance in SRA

- Compare standard SRA (continuous choice of price and quality) with
  - SRAs where one choice dimension is made dichotomous
  - Auctions where only either quality (FQA) or price (FPA) should be chosen

# Preview of the results

Our experimental results show:

- similar performance of FQA and SRA
- underbidding in SRA and overbidding in FQA

==> When a mechanism is more complex (i.e. larger number of dimensions over which a bidder has to make a choice, e.g. Payne (1976) and Enke and Zimmermann (2017)), bidders are more likely to make errors (i.e. play suboptimal strategies).

# This presentation:

- Literature
- Experimental Design
- Theory Predictions
- Experimental Results
- Quantal Response Equilibrium model
- Conclusions and policy implications

# Literature

- *Theory - Equilibrium conditions:*

Unidimensional case: Che (1993);
Multidimensional quasi-linear case: Branco (1999), Asker, Cantillon (2008)

- *Empirical*

Lewis and Bajari (2011): compare SRA with FPA
Cameron (2000): compares SRA with a more flexible mechanism

- *Experimental*

SRA: Bichler (2000); Albano et al. (2018).
Multiattribute: Chen-Ritzo et al.(2005); Wiggans et al.(2007); Strecker (2010)
Complexity: Payne (1976); Enke, Zimmermann (2017).

# Experimental Design

- 24 subjects in each session; 15 periods; in each period, subjects are randomly partitioned into 12 pairs. Pairs are formed within 4 re-matching groups of 6 subjects;
- For each of the 5 treatments, 3 sessions, 12 independent observations at the re-matching group level;
- Ztree; subjects are undergraduate students at Bocconi University, Milan; BELSS Lab.
- exchange rate: 1 euro per 7 tokens; initial budget for each subject: 20 tokens to cover the possibility of losses;
- on average, 14.47 euro for 70 minutes;
- Questionnaire (socio-demographics, risk aversion, joy of winning).

# Experimental Design

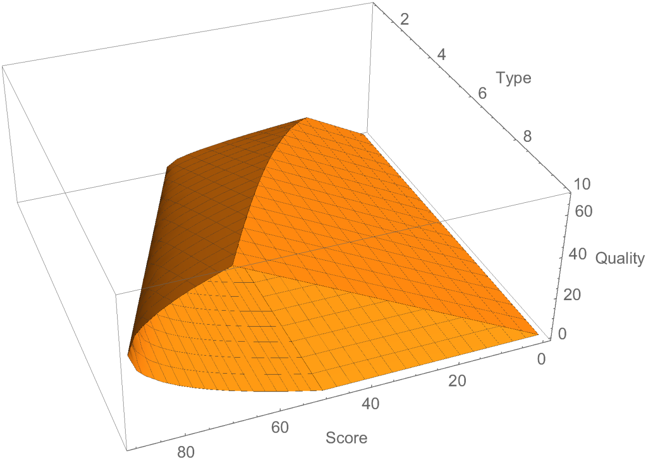
## Baseline Game

- Two sellers, auction to sell an object to a buyer.
- Bidimensional bid: quality $Q$ and price $p$
- Scoring rule auction (**SRA**): $s(Q, p) = 50 + 2Q - p$
  - *Scoring function maximizes ex-ante buyer's utility*
- Reserve price as a function of quality choice: $BWP(Q) = Q + 50$
- Quadratic cost function, 10 types $\theta$: $C(Q) = Q^2/4\theta$

- Buyer's utility function (unobserved by sellers): $U(Q, p) = \frac{20}{7}Q - p$

# Experimental Design

## SRA: tridimensional domain of all strategies



# Experimental Design

## Other treatments

Two treatments where the choice of one dimension is removed.

- First Price Auction (**FPA**): $\overline{Q} = 16$
- First Quality Auction (**FQA**): $\overline{p} = 32$

Two treatments where the choice of one dimension is made dichotomous, same scoring function as in SRA.

- **SRA2q**: $\overline{Q}_L = 9$, $\overline{Q}_H = 40$
- **SRA2p**: $\overline{p}_L = 12$, $\overline{p}_H = 65$

*All parameters maximize ex-ante buyer's utility.*

# Theory Predictions

**Total surplus**

- Reducing the multidimensionality of choices decreases total surplus (efficiency) of the treatment
  - One choice (price/quality) is reduced or made dichotomous in each treatment different than SRA.
- Restricting the choice of price has a lower impact on total surplus than restricting the choice of quality

**Bidding behavior**

- SRA will produce the highest score, followed by SRA2p, SRA2q, FQA and FPA.

# Experimental Results

## Welfare Analysis on Total Surplus: Descriptive Statistics.

| | FPA | FQA | SRA | SRA2p | SRA2q |
|---|---|---|---|---|---|
| Predicted relative efficiency | | | | | |
| Max | 0.61 | 0.79 | 0.91 | 0.89 | 0.88 |
| Observed relative efficiency | | | | | |
| Max | 0.609 | 0.834 | 0.823 | 0.805 | 0.801 |
| Nash | 0.978 | 1.072 | 0.904 | 0.935 | 0.973 |
| Observations | 180 | 180 | 180 | 180 | 180 |

This table reports mean of relative overall efficiency, by treatment, by using as a benchmark either the highest achievable overall surplus (Max) or the overall surplus associated with the Nash equilibrium (Nash). Re-matching group averages.

| | Max | Nash | Max | Nash |
|---|---|---|---|---|
| FPA | -0.224*** | -0.0939*** | -0.189*** | -0.138*** |
| SRA2p | -0.0290* | -0.137*** | -0.0650*** | -0.221*** |
| SRA2q | -0.0327** | -0.0996*** | -0.0508* | -0.200*** |
| SRA | -0.0111 | -0.168*** | -0.0632 | -0.288*** |
| trend | | | 0.00304** | -0.00562 |
| trend*treatm. | NO | NO | YES | YES |
| Constant | 0.834*** | 1.072*** | 0.813*** | 1.112*** |
| Observations | 900 | 900 | 900 | 900 |

One-way linear RE model: dependency within rematching group. Unit of observation is at the rematching level.

# Experimental Results

**Result 1** - *Differently from what theory predicts, total surplus produced by FQA and by SRA are equivalent. Moreover, FQA (and only FQA) performs significantly better w.r.t. Nash predictions.*

To explain Result 1, we move to investigate bidding behavior.

- Score is a comparable measure across all treatments
- But, different treatments have different theoretical performances
- We use the percent distance from the predicted (Nash) score
  - $score\_dist = (score\_observed - score\_theory)\,/\,score\_theory$

# Experimental Results

## Bidding Behavior: score - Descriptive Statistics

| | FPA | FQA | SRA | SRA2p | SRA2q |
|---|---|---|---|---|---|
| Score | 51.98 | 60.50 | 53.87 | 56.43 | 56.48 |
| Score: (Obs-Th)/Th | 0.00789 | 0.0442 | -0.144 | -0.0948 | -0.0595 |
| Observations | 1080 | 1080 | 1080 | 1080 | 1080 |

This table reports mean of bidders' score, observed and as % distance from predicted (Nash) levels, by treatment.

- *Overbidding: lower profit in case of winning, relative to equilibrium*
- *Underbidding: higher profit in case of winning, relative to equilibrium*

| score | All bids | | Winning bids | |
|---|---|---|---|---|
| FPA | -8.523*** | -9.324*** | -6.961*** | -8.311*** |
| SRA2p | -4.072*** | -9.567*** | -4.889*** | -8.806*** |
| SRA2q | -4.021*** | -8.976*** | -4.963*** | -8.552*** |
| SRA | -6.626*** | -13.68*** | -7.841*** | -13.38*** |
| trend | | -0.0452 | | -0.107 |
| trend*treatm | NO | YES | NO | YES |
| Constant | 60.50*** | 60.82*** | 67.61*** | 68.35*** |
| Observations | 5400 | 5400 | 2700 | 2700 |

Two-way linear RE models: potential individual dependency over repetitions and dependency within rematching group.

# Experimental Results

## Bidding Behavior: score distance - Parametric Analysis

| score_dist | All bids | | Winning bids | |
|---|---|---|---|---|
| FPA | -0.0363** | -0.0505*** | -0.0256 | -0.0318* |
| SRA2p | -0.139*** | -0.221*** | -0.103*** | -0.143*** |
| SRA2q | -0.104*** | -0.183*** | -0.0725*** | -0.115*** |
| SRA | -0.188*** | -0.299*** | -0.158*** | -0.228*** |
| trend | | 0.000452 | | 0.000164 |
| trend*treatm | NO | YES | NO | YES |
| Constant | 0.0442*** | 0.0410*** | 0.0725*** | 0.0713*** |
| Observations | 5400 | 5400 | 2700 | 2700 |

Two-way linear RE models: potential individual dependency over repetitions and dependency within rematching group.

# Experimental Results

**Result 2** - *Given the theoretical predictions, the observed bidding behavior shows underbidding in all the SRAs and overbidding in FQA. Trend reduces, but do not eliminate, underbidding in SRA.*

Departure from risk neutrality cannot explain underbidding in SRAs and overbidding in FQA.

The change in behavior occurs when we move from treatments with unidimensional choice to those with multidimensional choice:

- Could be related to the degree of complexity of the auction.
- Response time confirms this intuition
  - **Ranking**: **FPA - FQA - SRA2q - SRA2p - SRA**
  - From faster to slower, differences are all significant

# Quantal Response Equilibrium Model

- QRE provides an error structure to observed behavior
- Players make errors in a bounded rational way: the probability of playing a suboptimal strategy depends on the (relative) payoff associated with it.
- QRE model with logistic errors

$$\Pr(b) = \frac{\exp[d \times U(b;\theta)]}{\sum_{b \in B} \exp[d \times U(b;\theta)]}.$$

  - $d \geq 0$: error parameter ($d \to 0$ then random choice, $d \to \infty$, the payoff maximizing bid is chosen for sure)
  - $U(b;\theta)$ is a CRRA utility function, with Arrow-Pratt coefficient of relative risk aversion $= r$

# Quantal Response Equilibrium Model

| Treatment | d | r | -Log-Likelihood | N° strategies |
|---|---|---|---|---|
| FQA | 2.38 | 0.68 | 261.97 | 68 |
| FPA | 1.28 | 0.68 | 355.82 | 120 |
| SRA2q | 1.1 | 0.68 | 438.21 | 184 |
| SRA2p | 0.96 | 0.66 | 311.57 | 100 |
| SRA | 0.86 | 0.62 | 1376.21 | 1298 |
| FQA (no bin) | 2.66 | 0.67 | 527.06 | 259 |
| FPA (no bin) | 1.38 | 0.65 | 846.48 | 479 |

Bin: 4 unit interval in each continuous choice dimension presents in the treatment.

# Quantal Response Equilibrium Model

- $r$ : very similar across all treatments
- $d$ : capture all variation. Ranking:
  - unidimensional mechanisms first, then SRAs with discrete choice then SRA
  - equal to response time ranking, except FQA and FPA
  - within same number of choice dimensions, it matters: (i) type of choice $p$ vs $Q$ and (ii) strategy set dimension

# Conclusions

- There exists a trade-off between the awarding mechanism theoretical performance and its complexity (i.e. increasing in the number of dimensions over which the bidder has to submit an offer).
- When bidders are boundedly rational, higher complexity may lead to larger deviations from the theoretical prediction.
  - ==> the theoretical advantage (in terms of performance) of a more complex mechanism over a simpler one may vanish in practice.
- It does not exists an optimal mechanism for every situation: depending on the buyer's utility function and how bidders can deal with complexity, different awarding mechanisms may be appropriate
- Possible extension: the choice of the optimal scoring function is not a trivial problem (it even does not exists a general solution in the multidimensional case). What if also the buyer makes errors in designing the awarding mechanism?

# SUPPLEMENTARY MATERIAL

# Nash equilibrium bidding functions

We use $\alpha$ to denote a bidding strategies in the price dimension and $\beta$ for the quality dimension. $\theta$ is drawn from a continuous uniform distribution with support $[0, 10]$, and $p$ and $Q$ can be any positive number (up to the maximum admissible).

- *FPA*:
$$\alpha_1(\theta) = \begin{cases} 64 & \text{if} \quad \theta = 1 \\ 64(\theta - 1)^{-1} \ln \theta & \text{if} \quad \theta \in (1, 10] \end{cases} ;$$
- *FQA*:
$$\beta_2(\theta) = 8\sqrt{\theta + 1};$$
- *SRA*:
$$[\alpha_3(\theta); \beta_3(\theta)] = [4(3\theta - 1); 4\theta] ;$$

# Nash equilibrium bidding functions

- *SRA2p*:

$$[\alpha_4(\theta);\beta_4(\theta)] = \begin{cases} \left[12; 2\sqrt{6(\theta+1)}\right] & \text{if} \quad \theta \in [1,\hat{\theta}) \\ \left[65; \sqrt{\frac{130(\theta^2-\hat{\theta}^2)+(\hat{\theta}-1)\hat{b}^2}{\theta-1}}\right] & \text{if} \quad \theta \in [\hat{\theta},10] \end{cases},$$

where $\hat{\theta} = \left(65+30\sqrt{2}\right)/16$ and $\hat{b} = \left(59+15\sqrt{2}\right)/4$;

- *SRA2Q*:

$$[\alpha_5(\theta);\beta_5(\theta)] = \begin{cases} [81/4;9] & \text{if} \quad \theta = 1 \\ \left[81[4(\theta-1)]^{-1}\ln\theta; 9\right] & \text{if} \quad \theta \in (1,\hat{\theta}) \\ \left[\frac{400(\ln\theta-\ln\hat{\theta})+(\hat{\theta}-1)\hat{b}}{\theta-1}; 40\right] & \text{if} \quad \theta \in [\hat{\theta},10] \end{cases},$$

where $\hat{\theta} = 49/8$ and $\hat{b} = 62 + (162/41)\times\ln(49/8)$.

# Welfare Analysis on utility and profits: Descriptive Statistics.

| | Rel. Eff. | | Rel. Buyer Surp. | | Rel. Bidder Surp. | |
|---|---|---|---|---|---|---|
| | Max | Nash | Max | Nash | Max | Nash |
| FPA | 0.609 | 0.978 | 0.430 | 0.711 | 0.179 | 0.267 |
| FQA | 0.834 | 1.072 | 0.665 | 0.798 | 0.169 | 0.274 |
| SRA | 0.823 | 0.904 | 0.625 | 0.687 | 0.198 | 0.217 |
| SRA2p | 0.805 | 0.935 | 0.620 | 0.723 | 0.185 | 0.212 |
| SRA2q | 0.801 | 0.973 | 0.597 | 0.721 | 0.204 | 0.251 |

Note: this table reports mean of total surplus, buyer's utility and bidders' profits relative to maximum (i.e. first-best allocation) and predicted (i.e. Nash equilibrium) total surplus, by treatment.

# Welfare Analysis on buyer's utility: Parametric Analysis

| | Max | Nash | Max | Nash |
|---|---|---|---|---|
| FPA | -0.235*** | -0.0875*** | -0.216*** | -0.0629 |
| SRA2p | -0.0452* | -0.0754** | -0.0961** | -0.0792 |
| SRA2q | -0.0674*** | -0.0771** | -0.144*** | -0.155* |
| SRA | -0.0397 | -0.111*** | -0.0985 | -0.140 |
| trend | | | 0.00248* | 0.00785 |
| trend*treat | NO | NO | YES | YES |
| Constant | 0.665*** | 0.798*** | 0.647*** | 0.743*** |
| Observations | 900 | 900 | 900 | 900 |

One-way linear RE model: dependency within rematching group. Unit of observation is at the rematching level.

# Welfare Analysis on bidder's profit: Parametric Analysis

| | Max | Nash | Max | Nash |
|---|---|---|---|---|
| FPA | 0.0102 | -0.00638 | 0.0274 | -0.0755 |
| SRA2p | 0.0162 | -0.0614 | 0.0311 | -0.142 |
| SRA2q | 0.0347* | -0.0225 | 0.0930** | -0.0452 |
| SRA | 0.0285 | -0.0567 | 0.0353 | -0.148 |
| trend | | | 0.000555 | -0.0135 |
| trend*treat | NO | NO | YES | YES |
| Constant | 0.169*** | 0.274*** | 0.165*** | 0.368*** |
| Observations | 900 | 900 | 900 | 900 |

One-way linear RE model: dependency within rematching group. Unit of observation is at the rematching level.

# Bidding's Behavior: price and quality decision.

## Descriptive statistics

| | FPA | FQA | SRA | SRA2p | SRA2q |
|---|---|---|---|---|---|
| Price | 30.02 | 32 | 48.94 | 41.05 | 37.24 |
| Quality | 16 | 21.25 | 26.41 | 23.74 | 21.86 |
| Price: (Obs-Th)/Th | -0.00171 | 0 | 1.203 | 0.819 | 0.595 |
| Quality: (Obs-Th)/Th | 0 | 0.0609 | 0.308 | 0.202 | 0.111 |
| Observations | 1080 | 1080 | 1080 | 1080 | 1080 |

Note: this table reports mean of bidders' price and quality, observed and as % distance from predicted (Nash) levels, by treatment.

# Bidding's Behavior: price and quality decision.

## Disentangling price and quality decision in SRA

| | All bids | | Winning bids | |
|---|---|---|---|---|
| Price: (Obs-Th)/Th | | | | |
| my_parameter | | -0.321*** | | -0.105*** |
| Constant | 1.203*** | 2.918*** | 0.544*** | 1.243*** |
| Quality: (Obs-Th)/Th | | | | |
| my_parameter | | -0.0417*** | | -0.0135* |
| Constant | 0.308*** | 0.531*** | 0.215*** | 0.304*** |
| cov(e.p_dist x e.q_dist) | 0.836*** | 0.724*** | 0.264*** | 0.253*** |
| Observations | 1080 | 1080 | 540 | 540 |

Note: The table reports estimates from a SUR model with two-way linear random effects accounting for both potential individual dependency over repetitions and dependency within rematching group. Treatment is SRA. Covariance of the models'

# Response time: descriptive statistics

| | Response time |
|---|---|
| FPA | 26.85 |
| FQA | 34.68 |
| SRA | 96.78 |
| SRA2p | 70.62 |
| SRA2q | 60.12 |
| Observations | 5400 |

Note: this table reports mean of response time, in seconds and by treatment.

# Response time: parametric analysis

| Response time | | |
|---|---|---|
| FPA | -7.837** | -10.39** |
| SRA2p | 35.94*** | 38.87*** |
| SRA2q | 25.43*** | 31.97*** |
| SRA | 62.10*** | 55.58*** |
| trend | | -1.723*** |
| trend*treat | NO | YES |
| Constant | 34.68*** | 46.75*** |
| Observations | 5400 | 5400 |

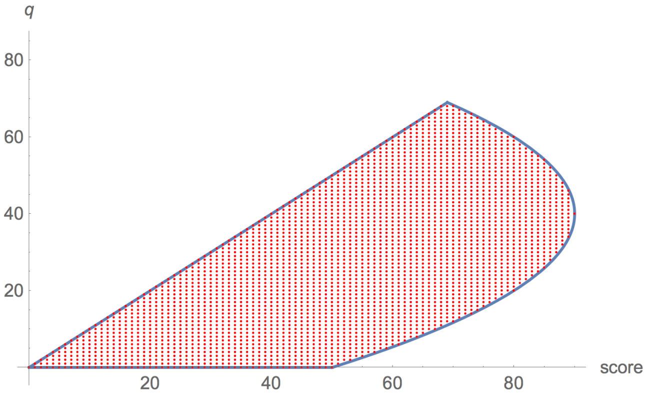
Note: The table reports estimates from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group.

# Binning methodology

- Binning is necessary for computational reason.
- To construct the binned strategies, we start from $q = 0$ and $s = 0$ and we assign to each bin $(q, s)$ all elements from $q$ to $q + 3$ and from $s$ to $s + 3$. We exclude all bins $(q, s)$ outside the domain of the feasible strategies.
- To compute utility for each binned strategy, we use the average score and the average quality for each bin: in other words we assign to a bin $(q, s)$ the utility provided by $(q + 1.5,\ s + 1.5)$, the only exception being for bins close to the boundary where the resulting average falls outside the domain of feasible bins.

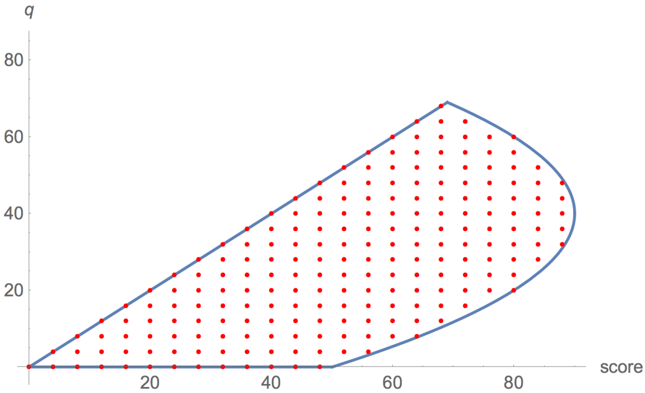
# Binning methodology EXAMPLE

Strategy set before binning; SRA; $\theta = 10$



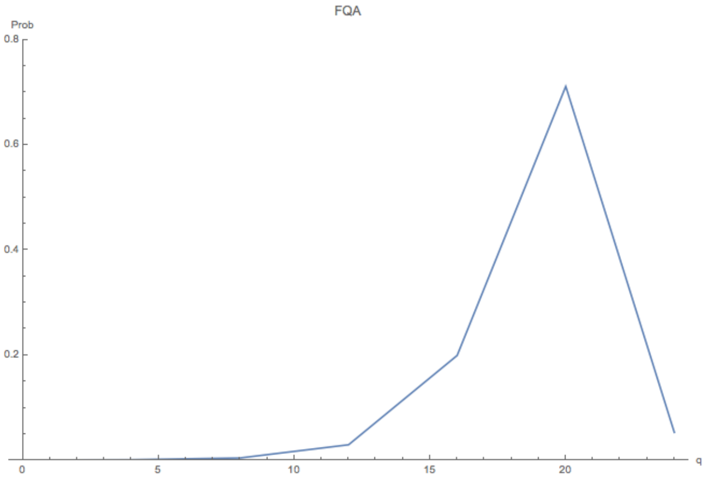
# Binning methodology EXAMPLE

Strategy set after binning; SRA; $\theta = 10$



# Probability of playing all possible strategies: FQA

$d = 2.38$; $r = 0.68$; $\theta = 5$.



# Probability of playing all possible strategies: SRA

$d = 0.86$; $r = 0.62$; $\theta = 5$.