Risk attitudes and optimal contracts: an experimental analysis

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1 Introduction

Contract theory has been one of the most active areas of research in economic theory since the last third of the XXth century. Significant progress has been achieved since then, especially in theoretic characteristics of the optimal schemes that stimulate the agent to act in the interests of the principal. However, theoretical optimal contracts are rarely used in practice, and this is for several reasons. One is relative complexity of optimal contracts, which makes them impractical in most real-life circumstances. Another is dependence of optimal contract from the specific utility functions of principals and agents. Many theoretical assumptions (e.g., quasilinear preferences, principal's risk neutrality and agent's risk aversion, knowledge about contracts parameters, etc.) need to be ensured to give full power to the optimal contract. By the same token, if any of these assumptions is not correct in practice, optimal contract shall be different from the canonical ones, precluding the possibility to test their predictions. In this project, we propose a direct experimental test of contract theory by simultaneous estimation of the utility function of the agents, their contractual decisions, and the contracts proposed by the principals. In our study, we combine a variety of common specifications for utilities (Moffatt, 2015) with theoretically efficient and empirical contracts proposed to and accepted by these agents (Hoppe and Kusterer, 2011; Henning-Schmidt e.a., 2010). If we are able to find a set of specifications for utilities and contract parameters which are consistent with each other and the data, then we do not reject the null hypothesis that contract theory based on traditional utilities works, at least in the in the laboratory settings. If, with all due diligence and efforts, we are unable to support the null – this will speak in favour of the alternative hypothesis that even in the clean laboratory settings the contracts are proposed and drawn on the principles which differ from those of contract theory. In either case, our results will constitute an important step towards understanding how theoretical models based on standard economics work in experimental (and hence, arguably, real – List and Fehr, 2015) settings (Lewitt and List, 2007; Al-Ubaydli e.a., 2017)

Over the past three decades, a large number of empirical works have been dedicated on the effectiveness of various incentive schemes. This literature is widely covered in the reviews of Prendergast (1999), Fehr and Falk (2003), Charness and Kuhn (2007), Gneeze (2011), Kőszegi (2014). The results of empirical laboratory and field experiments are ambiguous. On the one hand, laboratory test data help to identify cases in which compensation for effort has a stimulating effect. However, in a number of treatments, it is difficult to predict the outcome of compensation scheme working (Fehr and Falk (2002), Ariely (2007)). The proposed study is based on experimental testing of basic prerequisites of contract theory - agents preferences, which are common knowledge (including attitudes to risk), and rationality of principals and agents behavior.

A first component of our design is elicitation of risk preferences of the agents with standard tools, such as Multiple Price List methods (Holt and Laury, 2002). We estimate the parameters of the various standard utility functions of von Neumann-Morgenstern type, and assume, in line with the conventional contract theory, that these utilities characterize behavior of the agents in contractual relations. We test for theoretical consistency of utility and contract theories in a variety of settings, including playing against a simulated principal who offers optimal contracts given agent's preferences and simulated agent who play only if principal's offered fixed part of compensation covers the agent risk aversion, full information of the principals about risk preference parameters of the agents, and unstructured learning of both parties in

a repeated contractual task. In all cases, we allow for multiple periods and salient rewards to facilitate convergence of empirical behavior.

We specifically consider a linear contract, which is well spread in the real labor market. In order to better understand the nature of the enforcement of the contract by both principal and agent sides, we pursue the following research agenda: First, will we observe compensation standards, and test both fixed and piece rate parts of compensation on remoteness from optimality between the subjects. Second, will the compensation differ substantially between these subjects? Third, do we find positive correlation between risk preferences and risk covering by fixed salary? Finally, what is the nature of optimal / non-optimal behaviour? Will principal-agent couples, where one of the participants replaced by a robot, play similarly to pairs where both participants are people, and if not in what direction will the latter deviate from the "semi-optimal"?

2 Design

2.1 Base model

As a basic model, we take the standard moral hazard problem (Holmstrom and Milgrom, 1991). There are two players (principal and agent, denoted P and A). Principal's objective is to maximize B^p – her net gain, which depends on the agent's efforts (denoted $e \in R_+$) and the random variable $\varepsilon \sim N(0, \sigma^2)$ affecting payoff. The principal cannot verify the level of efforts of her agent, but she can observe the outcome (information signal) x which equals the sum of efforts and that random variable: $x = e_i + \varepsilon$. Following Holmstrom and Milgrom (1991), the principal offers her agent a linear contract which includes salary and piece-rate, and can be written as: $w(x) = a * x + \beta$. In the basic model, the principal's gain is represented by a linear function of the observed outcome x, and it can be written as x * s - w(x), where s is some positive multiplier ($s \in R_+$). Principal's net gain is $B^p = x * s - (a * x + \beta)$, which she strives to maximize by choosing the parameters of the linear contract.

Efforts for the agent are costly and denoted C(e), $(C(e) \in R)$ - a strictly convex function. If the contract is implemented, payoff of the agent, denoted as B^a , will be the difference between the contractual payment w(x), and the cost function C(e), in case of unacceptance of contract the agent will receive her reservation utility 0. When the value of the random variable ε is accounted for, payoffs to both agent and principal can be described in terms of the expected income. It is assumed that the principal is risk-neutral, since the expected income depends on a variety of agents, so her risk is diversified, and the utility function $U(B^p)$ can be described in terms of the mathematical expectation of the gain $U(B^p) = E(B^p)$, By contrast, the outcome to the agent depends only on the contract conditions, her chosen level of effort and the realization of the random variable. One way to describe the agent's preferences is to set the **CARA** utility function (constant absolute risk aversion), $U(w(x), C(e)) = 1 - e^{-r(w(x)-C(e))}$, where r is the coefficient of absolute risk aversion. Solutions for other utilities, such as CRRA, can be only numerical, and shall be implemented as such.

The agent solves her utility maximization problem $U(w, C, r) \to \max_{\{e\}}$. This creates conditions for solution of optimization problem. They are formed as inequalities, known in the literature as individual rationality and incentive compatibility (IC and IR). To do this, you need to consider the optimal level of effort e^* and any different from him in the smaller side, which is denoted for \hat{e} . Constraints of individual rationality $IR : \forall E[U(w(e+\varepsilon) - C(e))] \geq \underline{U}$ implies that it is not profitable for the agent to leave the game, because of her utility exceeds minimum level U. And constraints of incentive compatibility $IC : E[U(a^*(e^*+\varepsilon) + \beta^* - C(e^*))] \geq E[U(a^*(\hat{e}+\varepsilon) + \beta^* - C(\hat{e}))]$ implies that it is not advantageous for the agent to choose an effort level lower than given by the principal through the optimal compensation scheme $\{e^*, \beta^*, a^*\}$. If IR, IC, FOC and SOC conditions are met, then the solution of the problem of finding the optimal contract with a moral hazard exists.

2.2 Theoretical solution and specification of model for experiment

According to Holmstom and Milgrom 1991 we define $C(e) = e^2/2$, and the CARA utility function (constant absolute risk aversion), $U(w(x_i), C(e_i)) = 1 - e^{-r(w(x_i) - C(e_i))}$, where r is the coefficient of absolute risk aversion. The first-best solution can be obtained when public gain is maximized, because conditions is chosen so as to confiscate the entire rent from the agent (i.e. so (that constraint (IR) is satisfied with equality). By definition, the principal's profit without the risk is $B^p = e * s - (ae + \beta)$. Consequently FB utility of agent is $U(w) = a * e + \beta - 0.5 * e^2$, than maximization of $(e * s) - 0.5 * e^2 \rightarrow max_{[e]}$, generate the solution $e_{FB}^* = s/2$.

Further, lets consider second best solution under risk with distribution of $\varepsilon \sim N(0, \sigma^2)$. Denote risk premium as r_P For Principal's benefit changes only compensation for risk: $B^p = x * s - w(e) - r_P$, because of risk neutrality of principal: $u^p(x) = E(x) = e * s + E(\varepsilon * s) - C(e) - r_P$, where $E(\varepsilon * s) = 0$. Due to the exponential form of the agent's utility and agent risk aversion, the second best solution demands IC and IR in the following form:

$$E_{\varepsilon}\left[1 - \exp\left\{-r(w(x_i) - \frac{e_i^2}{2})\right\}\right] \ge 1 - \exp\{-r\underline{U}\}$$
(IR)

$$E_{\varepsilon} \left[1 - \exp\{-r(a^*(e^* + \varepsilon) + \beta^* - \frac{e^{*2}}{2})\}\right] \ge E_{\varepsilon} \left[1 - exp\{-r\left((a^*(\hat{e} + \varepsilon) + \beta^* - \frac{\hat{e}^2}{2}\right))\}\right]$$
(IC)

Here \underline{U} is gain that the agent will receive (with probability 1), refusing to enter into a contract. To find the solution of the principal-agent problem, it is necessary to find the optimal parameters a, β . From IR is inferred fixed part of compensation: $\beta = \underline{U} - e_i^2/2 + ra^2\sigma^2/2$.¹ and $r_P = r * a^2 * 0.5 * \sigma^2$. We can rewrite out the formula for the certainty equivalent of the agent $CE = a * e + \beta - C(e) - 0, 5 * ra^2\sigma^2$, i.e. the guaranteed amount that the agent will appreciate in the same way as the expected gain from the risky game.

Now the problem can be formalized in terms of social planner as $B^p = x * s - C(e) - 0, 5 * ra^2 \sigma^2 \rightarrow max$, (for e, a). Because of convexity, an optimal level of efforts in SB solution is lower than $e_{FB}^* = s/2$. From the condition IC follows directly that $a^* = e$, (Because according to IC for agent $a^* \in argmax(w(e)) =$ $[\beta + a * e - 0.5 * e^2 - 0.5 * r * a^2 * \sigma^2]$, finding from $\frac{\partial w(e) - c(e)}{\partial e} = 0$ generates solution $a^* = e$). The solution now depends only on efforts because optimal piece rate is equal to efforts ($a^* = e$) and β serves only for allocation of total gain. I.e. total gain only divides in a certain proportion the winning of the players since it does not depend on the fixed part of the compensation. Finally the principal needs to decide which level of effort maximizes her utility E_{ε} [$s * x - \alpha x - \beta$] = $a - a^2 - \underline{U} + \frac{a^2}{2} - \frac{ra^2\sigma^2}{2} = -\underline{U} + s * a - \frac{a^2}{2}(1 + r\sigma^2)$. From the first order condition, we obtain $e_{SB}^* = \frac{s}{1+r\sigma^2}$. What is obviously less than s/2 in case of $\sigma > 1$.

With the existing solution, it remains to determine the numerical values and the domain of some parameters. First we need to set s. We choose the value of the parameter to be equal to 10 according to Fehr and Schmidt (2004), who used a similar specification of the total gain function in experiments, also the number 10 is convenient for participants. Second, we need to impose limits to possible actions for participants-agents. It looks that middle point of FB optimal efforts (5 = s/2) is appropriate to outline the border of max effort as twice of risk-free solution, than $e \in (0, 10)$. It automatically implies restriction on $\beta \in (0, 100)$ as no more than max of e * s, and $a \in (0, 10)$ as no more than max efforts. Third we need to define $\sigma > 1$, we choose 4/3to distribute a random $\varepsilon \sim N\left[0, \frac{4}{3}\right]$, then the random variable x can take the values x(e) = (-3, 13) with more than 95 % probability, and x(e) = (-1, 11) with more than 50 % probability, which is also pretty intuitive for participant. All elements of the model specification are summarized in Table. 1

The last free parameter is risk aversion coefficient. We need that compensation scheme solutions for range of the risk aversion coefficient from 0 to 2 would be most of the subjects preferences are concentrated,

$${}^{1}E\left[e^{-r(\beta+a(e+\varepsilon)-0.5*e^{2}]}\right] = E\left[e^{-ra\varepsilon}\right] * e^{-r(\beta+a*e-0.5*e^{2})} , \text{ and in expressing } E\left[e^{-ra\varepsilon}\right] \text{ we obtain }$$

$${}^{\frac{1}{\sqrt{2\pi\sigma}}\int_{-\infty}^{\infty}e^{-ra\varepsilon}e^{-\frac{\varepsilon^{2}}{2\sigma^{2}}} = e^{-ra\sigma^{2}} * \frac{1}{\sqrt{2\pi\sigma}}\int_{-\infty}^{\infty}e^{-\frac{\varepsilon^{2}}{2\sigma^{2}}} = e^{-ra\sigma^{2}} , \text{ than } E\left[e^{-r(\beta+a(e+\varepsilon)-0.5*e^{2}]} = e^{-r(\beta+a*e-0.5*e^{2})-ra\sigma^{2}} , \text{ and finally from IR } e^{-r(\beta+a*e-0.5*e^{2})-ra\sigma^{2}} = e^{\underline{U}}. \text{ Solution for } \beta \text{ is } \beta = \underline{U} - 0.5*e^{2} + r*a^{2} * 0.5*\sigma^{2}$$



Figure 1: Dependence between degree of risk aversion and different part of compensation

were not only marginal (i.e. beta = 0), but also would be combinations of a and β . The chosen parameters just meet these conditions: Fig. 1.

Table 1. parameters summary							
The name of the variable (coefficient, parameter)	Value / Type	origin					
Agent utility	$U\left(w\left(x_{i}\right),C(e_{i})\right) = - \\ \mathrm{e}^{-r\left(w\left(x_{i}\right)-C(e_{i})\right)}$	Holmstrom Milgrom 1991					
C(e)	$0.5 * e^2$	Holmstrom Milgrom 1991					
ε	$\sim N(0,\sigma)$	Holmstrom Milgrom 1991					
		SB solution is stipulated					
$a*_{SB}$	e	by specification of $C(e)$, ε					
		and $U(w(x_i), C(e_i))$					
		SB solution is stipulated					
$\beta *_{SB}$	$\beta = \underline{U} - e_i^2/2 + ra^2\sigma^2/2.$	by specification of $C(e)$, ε and					
		$\mathrm{U}(w_{-}(x_{i}),C(e_{i}))$					
~*	a / 9	FB solution is stipulated					
e_{FB}	8/2	by specification of $C(e)$					
		SB solution is stipulated					
e^*_{SB}	$\frac{s}{1+r\sigma^2}$	by specification of $C(e)$, ε and					
	1,10	$\mathrm{U}(w_{-}(x_{i}),C(e_{i}))$					
S	10	For experimental design purpose					
effort domain	[0, 10]	For experimental design purpose					
σ	4/3	For experimental design purpose (must be >1)					
risk aversion	[0.3]	According to majority's risk					
coefficient domain	[0,3]	aversion in subjects pool					

Table 1: parameters summar

Optimal outcome with neutral risk preference is achieved in point 37.5, when $a = 5, \beta = 0, total_gain = 5 * 10 - 5^2 * 0.5$

Numerical example for people with risk aversion r = 1.5, $a \approx 3$, $e \approx 3$, $\beta \approx 6$. $w(x_i) = 3x_i + 6 \approx 15$, $C(e_i) = \frac{1}{2}e_i^2 = 4.5$, $B^p = x_i * s - a * e_i - \beta = 30 - 15 = 15$

2.3 Experimental procedure and design

Participants play a principal agent game with moral hazard. Principals were named 'Participant 1' and Agents 'Participant 2' in order to not induce framing effects. The parameter values are given in Table 1 in

the previous section.

mechanic of main game. Experiment contains several phases, where the main game (moral hazard) is included in second phase (information about all phases is available in table 3). In this game the participants perform the following sequence of actions: At the beginning the agent's level of risk aversion is shown to the principal, and he makes a decision about two options in proposed contract:

- 1. Fixed rate (β) , which takes values from 0 to 100
- 2. **Piece rate** (a), which takes values from 0 to 10

Next, the agent chooses the level of effort based on the information of costs and the parameters of proposed contract.

efforts	1	2	3	4	5	6	7	8	9	10
cost of efforts	0.5	2	4.5	8	12.5	18	24.5	32	40.5	50

Finally, a random value is realized and the participant's payoff in the round is calculated. Twenty rounds go through this way, (see table 3). In order to imitate the risk-neutrality of the principal, the principal's reward is taken in the form of Roth-Malouf (none monetary payoff but chance to get the sum of money in the end of the game). Thus Payoff of participants in each round is described by:

$$B^{A} = a * (e + \varepsilon) + \beta - C(e) \quad B^{P} = \begin{cases} 0, \text{ with probability } 1 - p \\ 20, \text{ with probability } p \end{cases}$$

where p is: $p = \frac{(e+\varepsilon)*s-(a*(e+\varepsilon)+\beta)}{25}$ Risk elicitation procedure. On the first phase, Participants make a decision about 10 choice option located in to different column A and B in MPL (Holt and Laury, 2002). The switching point between columns traces the risk aversion of the subject. Participants are informed that at the end of the experiment one of the lotteries will be chosen randomly and played. We chose MPL due to two reasons: first, we needed task with choice options wich generates variety of compensation payments, and for this, we needed to map the the risk aversion in utility functions from CRRA to CARA, which is shown in the table 2. Second, We needed to show participants the risk preferences of each other's. We used the verbal classification from the column "risk preference classification" in the table 2. An average choice by 3 rounds is showed to principals and than they can observe verbal classification to match agent actual choice and her risk attitudes.

Table 2: Mapping of risk aversion from CRRA to CARA utility functions

Number of safe choices	Range of relative risk aversion for $u(x) = \frac{x^{1-r}}{1-r}$	Range of absolute risk aversion for $u(x) = 1 - e^{-r*x}$	Risk preference classification	Optimal piece rate	Optimal salary
0-1	r<-0.95	r<-0.44	highly risk loving	5	0
2	-0.95 < r < -0.49	-0.44 < r < -0.21	very risk loving	5	0
3	-0.49 < r < -0.15	-0.21 <r<-0.01< td=""><td>risk loving</td><td>5</td><td>0</td></r<-0.01<>	risk loving	5	0
4	-0.15 < r < 0.15	-0.02 <r<0.< td=""><td>risk neutral</td><td>5</td><td>0</td></r<0.<>	risk neutral	5	0
5	0.15 < r < 0.41	0.< r < 0.35	slightly risk averse	5	0
6	0.41 < r < 0.68	0.35 < r < 0.573	risk averse	4.95	0.24
7	0.68 < r < 0.97	0.573 < r < 0.84	very risk averse	4	$3,\!98$
8	0.97 < r < 1.37	0.84 < r < 1.25	highly risk averse	3,08	5,9
9-10	1.37 <r< td=""><td>1.25 < r</td><td>stay in bed</td><td>0.2</td><td>1.27</td></r<>	1.25 < r	stay in bed	0.2	1.27

Simulated opponents. To test a theoretical consistency of utility and contract theories, we use the simulated robot opponents. In the instructions we inform the participants that there is a chance to play both against the robot and against the human. In each session we divide the subject pool into two, one group plays first round with the robot and the rest with the human, the rest vice versa: first round with the person the rest with the robot. At the same time participants do not know in which part they are.

Robots are programmed to play as follows: The robot-principal receives the risk aversion coefficient and calculates the parameters of the optimal contract, for each option from the MRL according to the last columns in the table 2, because salary could not be negative, we consider all cases with rj0.521 as risk neutral. The robot-agent chooses the level of effort equal to the piece rate if the proposed salary (insurance) is more than the optimal (also in table 2), otherwise chooses zero. Risk aversion coefficients of robots-agents are assigned according to participants' coefficients.

The general timeline of experiment. First, participants read instructions to phase 1, then play phase 1. After MPL they read second instructions to phase 2 and pass a quiz (we share instructions to avoid manipulating the choice as a signal of risk preferences for the principal). After that, they play phase 2. At the end of the experiment, participants go through a questionnaire for general questions and a questionnaire for revealing tendency to maximizing / optimizing. At the end of experiment everyone gets their payouts. Timeline information is summarized in the table 3.

Treatment % of subjects		Phase 1		Phase 2				
label		Action	Num of rounds		Num of rounds			
				Step 1	Step 2	Step 3		
$H_p vsH_a$	50%	Elicitation using Multiple Price List (MPL) by Laury and Holt (2002)	3	Information about average risk choice of agent from Phase 1 is showed to Pr., than Pr. offers the contract.	Ag. receives the contract and choose the efforts	Random variable is realized , everyone discovers	20	
$R_p vsH_a$	25%		3	Robot calculates optimal contract based on risk aversion of human participant elicited from Phase 1	Ag. receives the contract and choose the efforts	round's payoff	20	
$H_p vsR_a$	25%		3	Information about average risk choice of agent from Phase 1 is showed to Pr., than Pr. offers the contract.	Robot compares proposed salary to optimal and if proposed $\beta > \beta^*$ choose the efforts equal to proposed piece rate, else choose 0		20	

Table 3: Description of design phase

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